Old, frail, and uninsured: Accounting for features of the U.S. long-term care insurance market *

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Abstract

Half of U.S. 50-year-olds will experience a nursing home stay before they die, and one in ten will incur out-of-pocket long-term care expenses in excess of $200,000. Surprisingly, only about 10% of individuals over age 62 have private long-term care insurance (LTCI). This paper proposes a quantitative equilibrium optimal contracting model of the LTCI market that features screening along the extensive margin. Frail and/or poor risk groups are offered a single contract of no insurance that we refer to as a rejection. According to our model, rejections are the main reason that LTCI take-up rates are low. Both supply-side frictions due to private information and administrative costs and demand-side frictions due to Medicaid play important and distinct roles in generating rejections and the pattern of low take-up rates in the data.

Keywords: Long-Term Care Insurance; Medicaid; Adverse Selection; Insurance Rejections.

JEL Classification numbers: D82, D91, E62, G22, H30, I13.

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1 Introduction

One of the central questions in economics is how the presence of asymmetric information about an individual's risk exposure distorts insurance arrangements. Since the seminal work of Rothschild and Stiglitz (1976), the focus of theoretical and empirical research has been on the pricing and coverage of insured individuals.[1] However, recent research by Hendren (2013) and Chade and Schlee (2016) argues that private information has an important effect on the extensive margin, that is, who is offered insurance and who is denied coverage.[2] These papers describe settings where the private information distortion is so severe that there are no gains from trade between an insurer and all individuals in a particular risk group. They refer to this no-trade result as a denial or rejection because the optimal menu for the risk group is a pooling contract with no coverage. Our objective is to investigate the quantitative significance of the extensive margin in the U.S. market for long-term care insurance (LTCI) using a quantitative model. In the model, the insurer decides which risk groups to offer insurance to, as well as, pricing and comprehensiveness of coverage for insured risk groups. We use the model to show that denials are the central screening device in the U.S. LTCI market.

We choose to analyze the LTCI market because there is evidence of both adverse selection and insurance denials in the market. Finkelstein and McGarry (2006) provide empirical evidence using Health and Retirement Study (HRS) data that individuals have private information about their nursing home (NH) entry risk and that they act on it. Those who believe that the risk of a NH entry is high are more likely to purchase LTCI. Industry surveys find that 20% of LTCI applications are withdrawn or rejected by underwriters. We apply industry medical underwriting guidelines to HRS data and estimate that approximately 36% to 56% of individuals would be rejected by insurers if they applied for LTCI coverage between ages 55 and 66, the most common ages of application.

We consider the problem of a monopolist insurer who faces a group of risk-adverse individuals that are identical to the insurer but have one of two different private risk exposures, as analyzed by Stiglitz (1977). This model exhibits the classic result that the optimal menu consists of two contracts. One contract offers full coverage and the other contract offers partial or possibly even zero coverage at a lower premium. High-risk types self-select into the full coverage contract while low-risk types prefer the partial coverage contract. It follows immediately that screening along the extensive margin does not occur in this setting because at least one type (the high-risk type) is always insured. We then use two different mechanisms to activate extensive-margin screening and produce rejections of all members of a risk group. First, we model publicly provided LTCI.[3] Medicaid offers free means-tested

[1] See, for example, Hellwig (2010) Chade and Schlee (2012), Lester et al. (2015), Finkelstein and McGarry (2006), Chiappori and Salanie (2000) and Fang et al. (2008).

[2] Hendren (2013) shows one way to generate no-trade contracts and provides empirical evidence that the dispersion of private information is large in rejected risk groups. Chade and Schlee (2016) conduct a theoretical analysis that shows how administrative costs can produce optimal contracts that feature no-trade.

[3] A number of papers have documented signification interactions between public insurance and private insurance. For instance, Mahoney (2015), finds that U.S. bankruptcy laws provide implicit insurance against large health expenses, Fang et al. (2008) document evidence of advantageous selection in the the U.S. Medigap market and Brown and Finkelstein (2008) find that Medicaid crowds out the private market for LTCI.

NH insurance but is a secondary payer for NH claims. We show that if the benefits offered by Medicaid are sufficiently generous, there is no basis for trade and the optimal menu of private insurance consists of a single contract that provides no coverage to the entire risk group. Second, we model administrative costs. These costs reduce profits from insuring a given risk group and, if they are large enough, there again is no basis for trade.

Representative policies in the U.S. LTCI market only provide partial coverage against long-term care (LTC) risk and charge premia that are well above actuarially-fair levels. We show that optimal menus in our model also exhibit partial coverage even for high-risk types when Medicaid and/or administrative costs are present. In the case of Medicaid, if high-risk individuals face uncertainty about whether they will qualify for the means-test at the point of NH entry, they prefer a partial coverage contract to full coverage. Consequently, both contracts in the optimal menu can exhibit partial coverage. Variable administrative costs increase the slope of the isoprofit schedules and this can also produce optimal menus in which both contracts only cover a fraction of the loss.

Only 10% of those 65 and over hold LTCI policies. Take-up rates are declining in frailty, a noisy indicator of NH entry risk, and increasing in wealth. To capture these observations in our model we introduce cross-sectional variation in wealth and frailty and assume that they are observable by the insurer. To generate low LTCI take-up rates, the standard model requires that the fraction of high-risk individuals be so large that the insurer cannot make profits insuring both risk types. He responds by offering an optimal menu that features two contracts: a contract with positive insurance but at a price that makes it only attractive to the high-risk type and a contract that offers no coverage. The low-risk types choose the no-coverage contract and the LTCI take-up rate of the risk group is less than 100%. Thus, from the perspective of the standard theory of adverse selection, low LTCI take-up rates are due to choice. Our model allows for this possibility, but, also introduces the possibility that LTCI take-up rates are low because there is no basis for trade with frail and/or poor risk groups. In this regard, it is important to note that both Medicaid and administrative costs can produce variation along the extensive margin that is consistent, at least at a qualitative level, with the cross-sectional pattern of LTCI take-up rates in our data. Medicaid produces higher rejection rates among less wealthy individuals because they are more likely to qualify for Medicaid benefits and among frail individuals because they tend to have low wealth. Administrative costs result in rejections of frail risk groups because they have higher dispersion in private information and low wealth individuals because they tend to be more frail.

To determine the quantitative significance of contracts that feature no-trade, the model needs to be parametrized using data. We set the parameters that govern the scale of public means-tested NH benefits to reproduce benefit levels and take-up rates of Medicaid. Administrative costs in the model are chosen to reproduce industry averages of fixed and variable costs. The distribution of private information and in particular the role of choice versus rejections in the model is identified from data on self-reported NH entry probabilities, LTCI take-up rates, and NH entry rates.

In our parametrized model, low LTCI take-up rates are almost entirely due to variation along the extensive margin. Only 0.11% of all individuals are offered an optimal menu that provides the option of positive or no insurance, and choose no insurance. Thus, the reason that 90% of individuals don't own a LTCI policy in the model is because there is no basis for trade between them and the insurer. Accounting for the overall measured dispersion in self-reported NH probabilities plays a central role in this finding. Parametrizations of the model that assign a bigger role to choice menus produce too little dispersion in private information as compared to the data.

Results that emerge from our parametrized model have implications for a range of previous findings in the literature. For instance, our result that choice menus play essentially no role in accounting for low LTCI take-up rates has a direct bearing on the efficacy of the correlation test for adverse selection proposed by Chiappori and Salanie (2000) when applied to LTCI take-up rates and NH entry rates as in Finkelstein and McGarry (2006). If low LTCI take-up rates were primarily due to choice, as the standard model of adverse selection predicts, holders of LTCI would have higher NH entry rates than non-holders. However, in virtually all risk groups in our model either both high and low risk types are insured or neither type is insured. Thus, LTCI ownership rates are uncorrelated with NH entry rates in all but a tiny fraction of risk groups. Indeed, our model with a single source of private information produces the same observations that led Finkelstein and McGarry (2006) to conclude that multiple sources of private information are required to understand the U.S. LTCI market. Individuals in our model only have private information about NH entry risk and yet the model produces a small correlation between LTCI ownership and NH entry. Thus, a small correlation between insurance ownership and loss occurrence is actually consistent with a single source of private information when the extensive insurance margin is active.

Because we model Medicaid, our parametrized model is able to produce the low LTCI take-up rates of poor individuals. This result is related to Brown and Finkelstein (2008) who consider the impact of Medicaid on the demand for LTCI in a setting with exogenously specified insurance contracts and find that individuals in the bottom two-thirds of the wealth distribution don't purchase a full insurance actuarially-fair product when Medicaid NH benefits are available. Our strategy of modeling the issuer's problem creates new interactions between Medicaid and private LTCI. When Medicaid is present, not only do individuals prefer private insurance contracts that feature partial coverage, but private insurance is cheaper and the issuer's profits are lower. Since the issuer customizes pricing and coverage to fit the needs of each risk group, the crowding out effect of Medicaid is much smaller.

Ameriks et al. (2016) find that more affluent individuals are not interested in purchasing the set of LTCI policies available to them in the market, but would be interested in purchasing an ideal LTCI product. They refer to their result as the LTCI puzzle. Our results suggest that private information and administrative costs are important reasons for this puzzle. These two mechanisms allow our parametrized model to account for the low LTCI take-up rates of affluent individuals in the data. If either one of these mechanisms is turned off, the take-up rates of affluent individuals in the model substantially increase.

Finally, in the parametrized model, the dispersion of private information is higher in frail and poor risk groups and these groups are more likely to be offered a no-trade menu. This result is consistent with the empirical evidence in Hendren (2013) that adverse selection is more severe in groups of individuals that are more likely to be rejected by private LTC insurers.

The remainder of the paper is organized as follows. Section 2 provides an overview of the U.S. LTCI market. Section 3 presents the model. Section 4 describes identification and parametrization of the quantitative model. Section 5 assesses the ability of the model to reproduce non-targeted moments. Section 6 contains our main results and robustness analysis, and our concluding remarks are in Section 7.

2 The U.S. LTCI market

In the U.S. LTCI is primarily used to insure against lengthy NH stays. For this reason we focus on NH stays that exceed 100 days.[4] We estimate that the lifetime probability of a long-term NH stay is 30% at age 50.[5] On average, those who experience a long-term stay spend about 3 years in a NH. According to the U.S. Department of Health and Human Services, NH costs averaged $225 per day in a semi-private room and $253 per day in a private room in 2016. Thus, it is not unusual for lifetime NH costs to exceed $200,000.

Given the extent of NH risk in the U.S., one would expect that the private LTCI market would be large. But, only 10% of individuals aged 62 and older in the HRS have private LTCI. Moreover, in 2000, private LTCI benefits only accounted for 4% of aggregate NH expenses, while the share of out-of-pocket (OOP) payments was 37%.[6] Favreault and Dey (2016) estimate that 10.6% of individuals will incur OOP LTC expenses that exceed $200,000 and Kopecky and Koreshkova (2014) find that the risk of large OOP NH expenses is the primary driver of wealth accumulation during retirement.

Most LTCI is purchased from agents or brokers by individuals aged 55–66 years, while the average age of NH entry is 83.[7] At the time of applying for LTC coverage, applicants are asked detailed questions about their health status and financial situation. Some common questions include: Do you require human assistance to perform any of your activities of daily living? Are you currently receiving home health care or have you recently been in a NH? Have you ever been diagnosed with or consulted a medical professional for the following: a long list of diseases that includes diabetes, memory loss, cancer, mental illness, and heart disease? Do you currently use or need any of the following: wheelchair, walker, cane, oxygen? Do you currently receive disability benefits, social security disability benefits, or Medicaid?[8] Applicants are also queried about their income and wealth and asked to explain the specific source of resources that will be used to pay premia. Applicants are warned that premia increases are common and queried about their ability to cope with future premia increases. Finally, applicants are informed that, as a rule of thumb, LTCI premia should not exceed 7% of their income.[9]

Underwriting standards are strict and rejections are common. About 20% of formal applications are rejected via underwriting according to industry surveys (see Thau et al. (2014)). However, even prior to underwriting, insurance brokers screen out applicants. They discourage individuals from submitting a formal application if their responses indicate that they have poor health or low financial resources. Using HRS data, we estimate that 36%

[4]Another reason we focus on NH stays is because Medicare offers universal benefits for short-term rehabilitative NH stays of up to 100 days.

[5]In comparison, using HRS data and a similar simulation model, Hurd et al. (2013) estimate that the lifetime probability of having any NH stay for a 50 year old ranges between 53% and 59%.

[6]Source: Federal Interagency Forum on Aging-Related Statistics.

[7]Thau et al. (2014) report that only 10% of sales in 2013 were sold at work-sites.

[8]Source: 2010 Report on the Actuarial Marketing and Legal Analyses of the Class Program.

[9] Source: NAIC Guidance Manual for Rating Aspects of the Long-Term Care Insurance Model Regulation, March 11, 2005.

to 56% of 55–66 year olds would be rejected for LTCI if they applied based on health underwriting guidelines from Genworth and Mutual of Omaha.[10,11] Rejection rates are high even in the top half of the wealth distribution, ranging from 28% to 48%.

For individuals who are offered insurance, coverage is incomplete and premia are high. Insurers cap their losses by offering indemnities instead of service benefits. Brown and Finkelstein (2007) estimate that a "representative" LTCI policy in 2000 only covered about 34% of expected lifetime costs. Brown and Finkelstein (2011) find that coverage has improved in more recent years with a representative policy in 2010 covering 66% of expected lifetime costs.[12] More recently, Thau et al. (2014) report that policies that offer unlimited lifetime benefit periods have largely disappeared from the market. Brown and Finkelstein (2007) and Brown and Finkelstein (2011) also find that individual loads, which are defined as one minus the expected present value of benefits relative to the expected present value of premia, ranged from 0.18 to 0.51 (depending on whether or not adjustments are made for lapses) in 2000 and ranged from 0.32 and 0.50 in 2010. In other words, LTCI policies are sometimes twice as expensive as actuarially-fair insurance. Loads on LTCI are high relative to loads in other insurance markets. For instance, Karaca-Mandic et al. (2011) estimate that loads in the group medical insurance market range from 0.15 for firms with 100 employees to 0.04 for firms with more than 10,000 employees and Mitchell et al. (1999) estimate that loads for life annuity insurance range between 0.15 and 0.25.

Even though prices are high and coverage is incomplete, insurers have found that LTCI products are costly products to offer and profits have been low. In order to promote sales, brokers are given a substantial commission in the year that the policy is written and smaller commissions in subsequent years. In 2000, initial commissions averaged 70% of the first year's premium and, in 2014, they averaged 105%. However, total commissions over the life of a policy have been reasonably stable. They were about 12.6% of present-value premium for policies written in 2000 and 12.3% of present-value premium for policies written in 2014. Administrative expenses associated with underwriting and claims processing are also significant. These expenses averaged 20% of present-value premium in 2000 and 16% of present-value premium in 2014.[13] Finally, as pointed out in Cutler (1996), LTCI products are subject to intertemporal risk. These policies pay out, on average, about 20 years after they are written and if interest rates, retention rates or claims duration vary from an insurer's forecast the costs of the entire pool of policies changes. Insurers are under increasing pressure by regulators to provision for this risk by including a markup on the initial premium. The additional proceeds are held as reserves to provision against adverse future developments in claims.

Insurers have not been able to fully pass through higher costs to consumers. According to Cohen et al. (2013), most insurers have exited the market since 2003 and many insurers are experiencing losses on their LTCI product lines. New sales of LTCI in 2009 were below

[10] All HRS data work is done using our HRS sample. Details on our sample selection criteria are reported in Section 2 of the appendix.

[11] The rejection rate is 56% if we assume that all individuals who stated that they had ever been diagnosed with any of the diseases asked about are rejected and 36% if we assume that none of them are rejected.

[12] Most of this increase in coverage is due to the fact that the representative policy in 2010 includes an escalation clause that partially insures against inflation risk.

[13] These figures on costs are from the Society of Actuaries as reported in Eaton (2016).

1990 levels and, according to Thau et al. (2014), over 66% of all new policies issued in 2013 were written by the largest three companies.[14]

Our quantitative model should be consistent with these main features of the LTCI market. Namely, that take-up rates are low, rejections are common, coverage is incomplete, premia are high, and insurers' profits are low.

2.1 Adverse selection

One explanation for these observations that we pursue in the analysis that follows is adverse selection. Actuaries are keenly aware that the high costs of offering LTCI translate into high premia and that this in turn has a negative impact on the quality of the pool of applicants (see for instance Eaton (2016)). Academic research has also documented evidence of adverse selection in the LTCI market. Finkelstein and McGarry (2006) provide direct evidence of private information in the market. Specifically, they find that individuals' self-assessed NH entry risk is positively correlated with both actual NH entry and LTCI ownership even after controlling for characteristics observable by insurers. Hendren (2013) raises the possibility that Finkelstein's and McGarry's findings are driven by individuals in high risk groups. Specifically, he finds that self-assessed NH entry risk is predictive of a NH event for individuals who would likely be rejected by insurers. One objective of our analysis is to assess the quantitative significance of rejections. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we have repeated the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. In particular, we find evidence of private information at the 10 year horizon in a sample of individuals who would likely be rejected by insurers.[15]

Interestingly, even though Finkelstein and McGarry (2006) find evidence of private information in the LTCI market, they fail to find evidence that the market is adversely selected using the positive correlation test proposed by Chiappori and Salanie (2000). When they do not control for the insurer's information set, they find that the correlation between LTCI ownership and NH entry is negative and significant. Individuals who purchased LTCI are less likely to enter a NH as compared to those who did not purchase LTCI. When they include controls for the insurer's information set, they also find a negative although no longer statistically significant correlation. Finally, when they use a restricted sample of individuals who are in the highest wealth and income quartile and are unlikely to be rejected by insurers due to poor health they again find a statistically significant negative correlation.

2.2 Public LTCI

We also model the crowding out effect that public insurance, particularly Medicaid, has on the demand for private insurance. Medicaid provides a safety net for those who experience

[14]The top three insurers are Genworth, Northwest and Mutual of Omaha. For information about losses on this business line see, e.g., *The Insurance Journal*, February 15, 2016, `http://www.insurancejournal.com/news /national/2016/02/15/398645.htm` or Pennsylvania Insurance Department MUTA-130415826.

[15]See Section 2.3 of the appendix for more details.

long-term NH stays. It is means-tested and only available to individuals who have either low wealth and retirement income (categorically needy) or low wealth and very high medical expenses (medically needy). Medicaid is also a secondary payer that only offers benefits after any private LTCI benefits have been exhausted. Brown and Finkelstein (2008) have documented the pronounced crowding-out effects that Medicaid has on the demand for private LTCI. They find, for instance, that in the presence of Medicaid about two-thirds of individuals would not purchase an ideal private LTCI policy.

3 Modeling the market for LTCI

In this section we start by describing two distinct ways to generate low LTCI take-up rates in an adverse selection model: optimal menus that provide individuals with a choice of two contracts such that good-risks self-select into the contract that offers no insurance, and no-trade optimal menus in which the entire risk group is offered no insurance. We show that either administrative costs or public means-tested insurance are needed to produce the no-trade optimal menu. We also show that either mechanism can also produce partial coverage contracts for all insured individuals in a risk group under certain conditions. Having made these points, we then explain the additional details that are needed to make the model suitable for quantitative analysis.

The heart of our model is a variant of the Rothschild and Stiglitz (1976) model with risk adverse individuals and a single monopoly issuer as in Stiglitz (1977). The assumption of a single issuer is a parsimonious way to capture the concentration we documented above in this market.[16] We extend the model by adding administrative costs on the insurer and Medicaid. Our modeling of administrative costs is inspired by Chade and Schlee (2016) who conduct a theoretical analysis of administrative costs and no-trade menus in an adverse selection model with a continuum of private types. We are unaware of other work that incorporates a public means-tested insurer into an optimal contracting framework.

3.1 Optimal contracts with adverse selection and administrative costs

Suppose that there is a continuum of individuals and that each individual has a type $i \in \{g, b\}$. They each receive endowment ω but face the risk of entering a NH and incurring costs m. The probability that an individual with type i enters a NH is $\theta^i \in (0, 1)$. A fraction $\psi \in (0, 1)$ of individuals are good risks who face a low probability θ^g of a NH stay. The remaining $1 - \psi$ individuals are bad risks whose NH entry probability is $\theta^b > \theta^g$. Let η denote the fraction of individuals who enter a NH then $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$. Each individual observes his true NH risk exposure but the insurer only knows the structure of uncertainty. A contract consists of a premium π^i that the individual pays to the issuer and an indemnity ι^i that the issuer pays to the individual if he incurs NH costs m. A menu consists of a pair of contracts (π^i, ι^i), one for each private type $i \in \{g, b\}$.

[16] Lester et al. (2015) propose a framework with adverse selection that allows one to investigate how optimal contracts vary with the extent of market power. However, for reasons of tractability they assume risk neutrality and their optimal contracts are different from ours.

The optimal menu of contracts offered by the insurer maximizes his profits subject to participation and incentive compatibility constraints. Our specification of the insurer's profits includes two administrative costs. The first cost is a variable cost of paying claims with constant of proportion $\lambda - 1 \geq 0$ and the second cost is a fixed cost $k \geq 0$ of paying claims. Thus, profits are

$$\psi\Big\{\pi^g - \theta^g\big[\lambda\iota^g + kI(\iota^g > 0)\big]\Big\} + (1-\psi)\Big\{\pi^b - \theta^b\big[\lambda\iota^b + kI(\iota^b > 0)\big]\Big\}. \quad (1)$$

This formulation is general enough to handle the various costs incurred by insurers that we described in Section 2. In Section 1.4 of the appendix, we show that costs that are proportional to premia such as brokerage fees and pricing margins can be mapped into the variable cost term and that costs of underwriting are captured by the fixed cost. The participation and incentive compatibility constraints for each type are

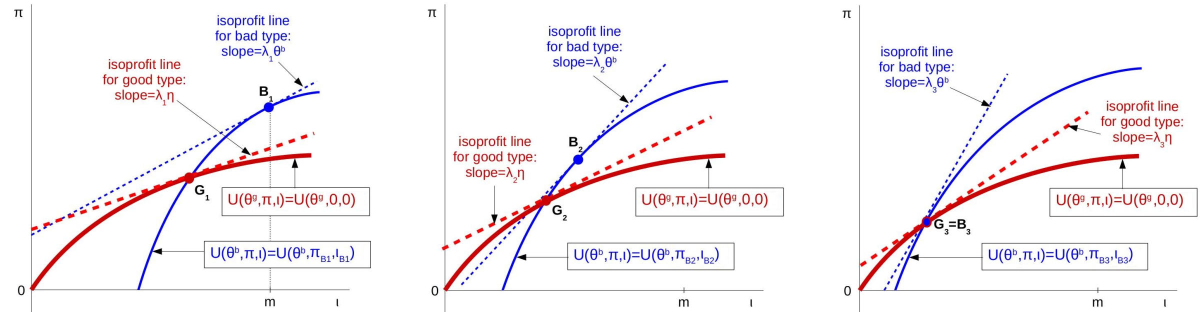
$$(PC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, 0, 0) \geq 0, \quad i \in \{g, b\}, \quad (2)$$

$$(IC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, \pi^j, \iota^j) \geq 0, \quad i, j \in \{g, b\},\ i \neq j, \quad (3)$$

where $U(\theta^i, \pi^i, \iota^i) = (1-\theta^i)u(\omega - \pi^i) + \theta^i u(\omega - \pi^i - m + \iota^i)$ is the utility of an individual with NH entry probability θ^i who chooses contract (π^i, ι^i). Individuals choose the contract from the menu that maximizes their utility. The participation constraints ensure that each type of individual prefers the contract designed for his type over no insurance and the incentive compatibility constraints ensure that each type prefers his own contract over the other types contract.

Figure 1a illustrates a typical optimal menu under the standard case: $\lambda = 1$ and $k = 0$. The menu exhibits the classic properties of an optimal menu under adverse selection. Specifically, the menu features two distinct contracts. The bad types prefer the contract at point B_1 that features full coverage against the loss and the good types prefer the contract at point G_1 which exhibits partial coverage, $0 \leq \iota^g < m$, but a smaller premium, $\pi^g < \pi^b$. Note that pooling contracts cannot be equilibria in this setting because starting from a pooling contact at a point such as G_1, the insurer can always increase total profits by offering the bad types a more comprehensive contract. Note also that the participation constraint binds for the good risk types, while the incentive compatibility constraint binds for the bad risk types. The contracts generally feature cross-subsidization from good to bad types. However, a separating equilibrium where the good types have a $(0, 0)$ contract can occur if the fraction of good types, ψ, is sufficiently low and the dispersion in the θ's is sufficiently high. This particular type of optimal menu is important because it is the only way for the standard model to produce a LTCI take-up that is less than one. In this equilibrium, all individuals are offered positive insurance, but the good risk types choose the $(0, 0)$ contract. From this we see that the extensive margin does not operate in the standard setup. Lack of insurance can only arise via choice menus and the LTCI take-up rate is always strictly positive because there is always trade with the bad risks.

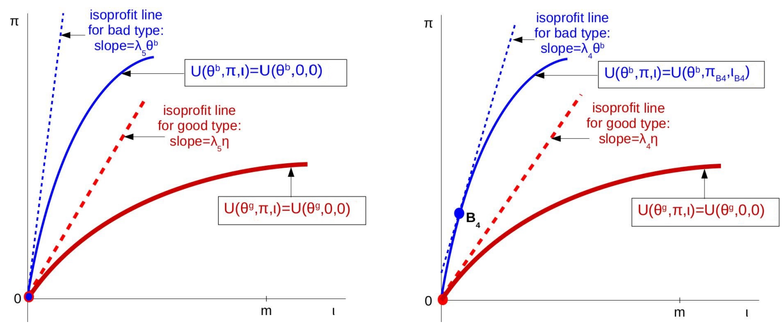
Allowing for variable administrative costs has a big impact on the properties of the optimal menu. With $\lambda > 1$, the optimal menu exhibits less than full insurance for both risk types. Pooling contracts can arise and when the costs are sufficiently large the extensive margin becomes active and the entire risk group is rejected. The various types of optimal



(a) Separating equilibrium with $\lambda = 1$

(b) Separating equilibrium with $\lambda > 1$

(c) Pooling equilibrium with $\lambda > 1$



(d) No trade equilibrium with $\lambda > 1$

(e) Only bad types have insurance with $\lambda > 1$

Figure 1: An illustration of the effects of increasing the insurer's proportional overheard costs factor (λ) on the optimal menu. The blue (red) lines are the indifference curves of bad (good) types. The dashed blue lines are isoprofits from contracts for bad types and the red dashed lines are isoprofits from a pooling contract.

menus that can arise with variable administrative costs are displayed in Figure 1. Start by by comparing Figure 1a with Figure 1b which shows an optimal separating menu when λ is above 1.[17] Increasing λ increases the slopes of the issuer's isoprofit lines. The issuer responds by reducing indemnities and premia of both types and the optimal contracts move southwestward along the individuals' indifference curves. Thus if $\lambda > 1$, the property of the standard model that bad types get full insurance no longer holds as both types are now offered contracts where indemnities only partially cover NH costs.[18]

Since the marginal costs of paying out claims to the bad type are higher than to the good type, when λ increases, the contracts also get closer together and a single (pooling) contract may arise. Figure 1c depicts such a case where both types get the same nonzero contract. Once a pooling contract occurs, the equilibrium under any larger values of λ will also involve pooling. However, the pooling contract will be lower down on the good types' indifference curve and feature less coverage, lower premia, and lower profits. If λ is sufficiently large

[17]Note that the good types' contract in the figure is illustrated as the optimal pooling contract. Conditions under which this holds, as well as, conditions characterizing the optimal contracts in the general case are provided in Section 1.1 of the appendix.

[18]See Proposition 1 in Section 1.1 of the appendix for a formal proof of this claim.

then no profitable nonzero pooling contract will exist. Figure 1d illustrates this no-trade case where the optimal menu consists of a pooling $(0, 0)$ contract and the LTCI take-up rate is zero.[19] Note that, as in the standard case, choice menus where only the bad risk types have positive insurance, such as the one depicted in Figure 1e, can also occur when $\lambda > 1$. Thus, with administrative costs, a risk group's LTCI take-up rate can be less than one due to either choice or no-trade menus.

Fixed administrative costs, k, can also produce rejections. Increasing k reduces the profits of non-zero contracts and the isoprofit schedules of each type i shift down in a parallel fashion for $\iota^i > 0$. If k is sufficiently high, no profitable menus may exist, resulting in no trade. However, k does not affect the properties of optimal menus featuring positive insurance that remain profitable.

3.2 Optimal contracts in the presence of Medicaid

Medicaid can also produce optimal menus that exhibit partial coverage for both types and no-trade equilibria. To establish how and when this occurs assume for the time being that there are no administrative costs ($\lambda = 1$, $k = 0$). Suppose, instead, that individuals who experience a NH event receive means-tested Medicaid transfers according to

$$TR(\omega, \pi, \iota) \equiv \max\left\{0, \underline{c}_{NH} - [\omega - \pi - m + \iota]\right\}, \tag{4}$$

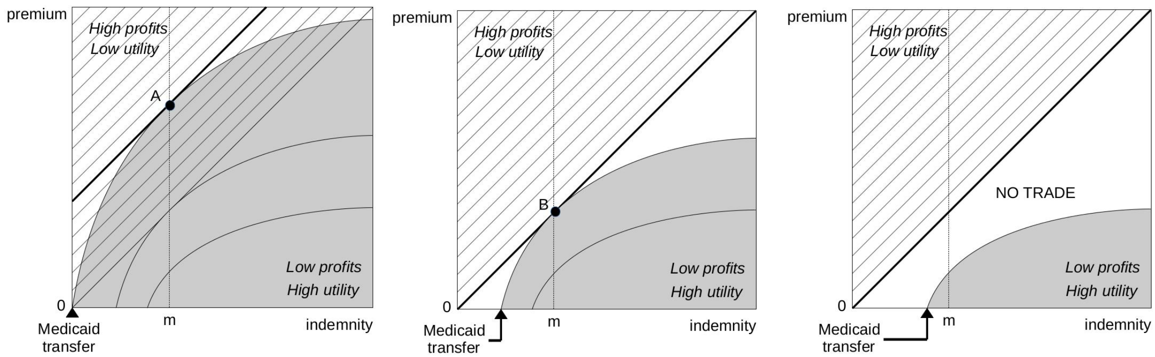
where $\underline{c}_{NH}$ is the consumption floor. Then consumption in the NH state is

$$c^i_{NH} = \omega + TR(\omega, \pi^i, \iota^i) - \pi^i - m + \iota^i. \tag{5}$$

By providing NH residents with a guaranteed consumption floor, Medicaid increases utility in the absence of private insurance thus reducing demand for such insurance. Moreover, Medicaid is a secondary payer. When $\underline{c}_{NH} > \omega - \pi - m + \iota$, marginal increases in the amount of the private LTCI indemnity ι are exactly offset by a reduction in Medicaid transfers and individual utility remains constant at $u(c_{NH}) = u(\underline{c}_{NH})$. It follows that the marginal utility of the insurance indemnity is zero for individuals who meet the means-test and only non-zero LTCI contracts that satisfy $\iota - \pi > \underline{c}_{NH} + m - \omega$ are potentially attractive to them.

Suppose that without Medicaid, the optimal contract of one of the types is given by point A in Figure 2a. Figure 2b illustrates the impact of introducing Medicaid with a small value of $\underline{c}_{NH}$. Notice that the optimal indemnity is unchanged. However, the individual's outside option has improved, and to satisfy the participation constraint, the premium is reduced. Because the insurer gives the individual the same coverage at a lower price, his profits decline. As $\underline{c}_{NH}$ increases, an equilibrium, such as the one depicted in Figure 2c, will eventually occur. In this case, $\underline{c}_{NH}$ is so large that the insurer can not give the agent an attractive enough positive contract and still make positive profits. The optimal contract is a no-trade, $(0, 0)$, contract. From this we see that, like the fixed administrative costs k, Medicaid reduces profits for the insurer but does not impact the extent of coverage conditional on positive insurance. Thus, if bad types are offered positive insurance they will receive full coverage

[19] Proposition 2 in Section 1.1 of the appendix provides a set of necessary and sufficient conditions for no-trade menus.



(a) Non-binding consumption floor (b) Low consumption floor (c) High consumption floor

Figure 2: Illustrates the effects of Medicaid on the trading space. The straight lines are the insurer's isoprofit lines and the curved lines are the individual's indifference curves.

against the NH event.[20]

In practice, at the time of LTCI purchase, most individuals do not know how much wealth they will have at the time a NH event occurs and are, thus, uncertain about whether and to what extent Medicaid will cover their costs if they experience a NH stay. As we now illustrate, modeling this uncertainty affects the amount of coverage that individuals demand. To see this suppose that when individuals are choosing their LTCI contract, they face uncertainty about the size of their endowment. Specifically, let ω be distributed with cumulative distribution function $H(\cdot)$ over the bounded interval $\Omega \equiv [\underline{\omega}, \overline{\omega}] \subset \mathbb{R}_+$ with $\underline{\omega} \geq m$ so that LTCI is always affordable. Then an individual's utility function is given by

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \left[\theta^i u(c^i_{NH}(\omega)) + (1-\theta^i)u(c^i_o(\omega))\right] dH(\omega), \tag{6}$$

where

$$c^i_o(\omega) = \omega - \pi^i, \tag{7}$$

$$c^i_{NH}(\omega) = \omega + TR(\omega, \pi^i, \iota^i) - \pi^i - m + \iota^i, \tag{8}$$

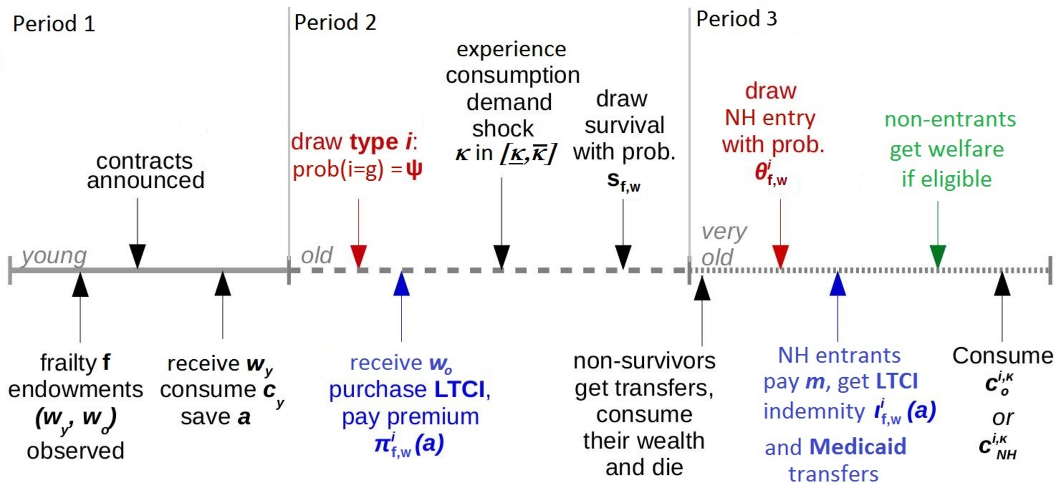
and the Medicaid transfer is defined by (4).

With random endowments, in the case of a NH event, an individual may only be eligible for Medicaid under smaller realizations of the endowment. A private LTCI product is thus potentially valuable because it provides insurance in the states of nature where the endowment is too large to satisfy the means-test. However, the individual will not want full private LTCI coverage because, due to Medicaid, he is already partially insured against NH risk in expectation.[21]

[20] This can occur in two different ways. First, if wealth is sufficiently high and bad types do not satisfy the means-test. Second, if the consumption floor provided by Medicaid is sufficiently low.

[21] See Section 1.2 of the appendix for a detailed discussion of this version of the model, and Propositions 3 and 4 which provide a sufficient condition for partial coverage contracts and a set of necessary conditions

Figure 3: Timeline of events in the baseline model.



3.3 The quantitative model

Our goal is to conduct a quantitative analysis of the LTCI market. In particular, we want to analyze how adverse selection, administrative costs, and Medicaid influence LTCI take-up rates, comprehensiveness of coverage, and pricing for groups of individuals who differ along two dimensions that are observable to insurers: frailty and wealth. We now describe how we extend our model to achieve this objective.

3.3.1 Individual's problem

In the U.S. most individuals purchase private LTCI around the time of retirement. Their saving decisions up to this point in time have been influenced, not only by their assessment of NH entry risk, but also by their assessment of the amount of public and private insurance they can obtain to help them cope with this risk. The distribution of wealth in turn influences the optimal contracting problem of the insurer. Those with high wealth have the outside option of self-insuring and those with low wealth have the outside option of relying on Medicaid if they experience a NH event. We capture the fact that wealth is a choice in a parsimonious way by dividing an individual's life into three periods. In period 1, he works and decides how much of his income to save for retirement.[22] In period 2, he retires, decides whether to purchase LTCI, and then experiences realizations of consumption demand and survival shocks. Finally, in period 3, he experiences a realization of the NH entry shock.

Figure 3 shows the timing of events in the model. At birth, an individual draws his frailty status f and lifetime endowment of the consumption good $\mathbf{w} = [w_y, w_o]'$ which are jointly distributed with density $h(f, \mathbf{w})$. Frailty status and endowments are noisy indicators of NH risk. He also observes his probability of surviving from period 2 to period 3, $s_{f,\mathbf{w}}$, which varies with f and $\mathbf{w}$, and the menus of LTCI contracts that will be available in period 2. A

for no-trade menus.

[22] In Section 1.5 of the appendix we show how our 3 period model could be easily mapped into a model that allows for more periods during individuals' working-age before LTCI purchase occurs.

working-aged individual then decides how to divide his earnings, w_y, between consumption c_y and savings a. This decision is influenced by Medicaid and also the structure of LTCI contracts. Medicaid benefits are means-tested and this creates an incentive to save less so that the individual can qualify for Medicaid. LTCI contracts vary with assets and this may induce individuals to save more if risk groups with higher assets face lower premia and/or more comprehensive coverage.

In period 2, the individual receives a pension w_o and observes his true risk of entering a NH conditional on surviving to period 3: $\theta^i_{f,\mathbf{w}}$, $i \in \{g, b\}$ with $\theta^g_{f,\mathbf{w}} < \theta^b_{f,\mathbf{w}}$. With probability ψ the individual realizes a low (good) NH entry probability, $i = g$, and with probability $1-\psi$ he realizes a high (bad) NH entry probability, $i = b$. The individual's true type $i \in \{g, b\}$ is private information. We assume that NH entry probabilities also depend on f and $\mathbf{w}$. The individual then chooses a LTCI contract from the menu offered to him by the private insurer.[23] The insurer observes and conditions the menu of contracts offered to each individual on their frailty status, endowments, and assets. We assume that the insurer observes assets because, as we discussed above, LTC insurers are required by regulators in many states to ascertain that the LTCI product sold to an individual is suitable (affordable).[24] Each menu contains two incentive-compatible contracts: one for the good types and one for the bad types. A contract consists of a premium $\pi^i_{f,\mathbf{w}}(a)$ that the individual pays to the insurer and an indemnity $\iota^i_{f,\mathbf{w}}(a)$ that the insurer pays to the individual if the NH event occurs.

After purchasing LTCI, individuals experience a demand shock that induces them to consume a fraction κ of their young endowment where $\kappa \in [\underline{\kappa}, \overline{\kappa}] \subseteq [0, 1]$ has density $q(\kappa)$. The demand shock creates uncertainty about the size of wealth at the time of NH entry and thus is important if the model is to attribute partial coverage to Medicaid as we explained above. More generally, it allows the model to capture the following features of NH events in a parsimonious way. On average, individuals have 18 years of consumption between their date of LTCI purchase and their date of NH entry during which they are exposed to medical expense and spousal death risks among other risks. In addition, the timing of a NH event is uncertain and individuals who experience a NH event later in life than others are likely to have consumed a larger fraction of their lifetime endowment beforehand.

Period 2 ends with the death event. With probability $s_{f,w}$ individuals survive until period 3 and with probability $1 - s_{f,w}$ they consume their wealth and die.[25] We model mortality risk because it is correlated with frailty and wealth, and impacts the likelihood of NH entry.

Finally, in period 3 the NH shock is realized and those who enter a NH pay cost m and receive the private LTCI indemnity. NH entrants may also receive benefits from the

[23] We assume the insurer does not offer insurance to working-age individuals in period 1 because LTCI take-up rates are low among younger individuals. For example, only 9% of LTCI buyers were less than 50 years old in 2015 according to LifePlans, Inc. "Who Buys Long-Term Care Insurance? Twenty-Five Years of Study of Buyers and Non-Buyers in 2015–2016" (2017).

[24] The reference in footnote 9 contains a model worksheet for reporting financial assets that is used to determine suitability. Lewis et al. (2003) reports that 31 States had adopted some form of suitability guidelines by 2002 and Chapter 5 of " Wall Street Instructors Long-term Care Partnerships online training course" `https://www.wallstreetinstructors.com/ce/continuing_education/ltc8/id32.htm` explains how suitability is assessed in the state of Florida.

[25] There is evidence that individuals anticipate their death. Poterba et al. (2011) have found that most retirees die with very little wealth and Hendricks (2001) finds that most households receive very small or no inheritances. This assumption eliminates any desire for agents to use LTCI to insure survival risk.

public means-tested LTCI program (Medicaid). Medicaid is a secondary insurer in that it guarantees a consumption floor of $\underline{c}_{NH}$ to those who experience a NH shock and have low wealth and low levels of private insurance.

An individual of type $(f, \mathbf{w})$ solves the following maximization problem, where the dependence of choices and contracts on h and w is omitted to conserve notation,

$$U_1(f, \mathbf{w}) = \max_{a \geq 0, c_y, \mathbf{c}_{NH}, \mathbf{c}_o} u(c_y) + \beta U_2(a), \tag{9}$$

with

$$U_2(a) = \left[\psi u_2(a, \theta^g_{f,\mathbf{w}}, \pi^g, \iota^g) + (1-\psi) u_2(a, \theta^b_{f,\mathbf{w}}, \pi^b, \iota^b)\right], \tag{10}$$

and

$$\begin{aligned} u_2(a, \theta^i, \pi^i, \iota^i) = \int_{\underline{\kappa}}^{\overline{\kappa}} \Big\{ u(\kappa w_y) + \alpha \Big[s_{f,\mathbf{w}} \big(\theta^i u(c^{i,\kappa}_{NH}) + (1-\theta^i) u(c^{i,\kappa}_o) \big) \\ + (1 - s_{f,\mathbf{w}}) u(c^{i,\kappa}_o) \Big] \Big\} q(\kappa) d\kappa, \end{aligned} \tag{11}$$

subject to

$$c_y = w_y - a, \tag{12}$$

$$c^{i,\kappa}_o + \kappa w_y = w_o + (1+r)a - \pi^i(a), \quad i \in \{g, b\}, \tag{13}$$

$$c^{i,\kappa}_{NH} + \kappa w_y = w_o + (1+r)a + TR(a, \pi^i(a), \iota^i(a), m, \kappa) - \pi^i(a) - m + \iota^i(a) \tag{14}$$

where α and β are subjective discount factors. The parameter β captures discounting between the time individuals enter the working-age and the time of retirement and the parameter α captures discounting between the time of retirement and the time of NH entry. The Medicaid transfer is

$$\begin{aligned} TR(a, \pi, \iota, m, \kappa) = \\ \max \left\{0, \underline{c}_{NH} - \left[w_o + (1+r)a - \kappa w_y - \pi - m + \iota\right]\right\}, \end{aligned} \tag{15}$$

and r denotes the real interest rate.

In the U.S. retirees with low means also receive welfare through programs such as the Supplemental Security Income program. We capture these programs in a simple way. After solving the agent's problem above which assumes that there is only a consumption floor in the NH state, we check whether they would prefer, instead, to save nothing and consume the following consumption floors: $\underline{c}_{NH}$ in the NH state and $\underline{c}_o$ in the non-NH state. If they do, we allow them to do so and assume that they do not purchase LTCI.[26]

[26]Modeling the Supplemental Security Income program in this way helps us to generate the low levels of savings of individuals in the bottom wealth quintile without introducing additional nonconvexities into the insurer's maximization problem.

3.3.2 Insurer's problem

The insurer observes each individual's endowments $\mathbf{w}$, frailty status f, and assets a. He does not observe an individual's true NH entry probability, $\theta^i_{f,\mathbf{w}}$, but knows the distribution of NH risk in the population and the individual's survival risk $s_{f,\mathbf{w}}$. We assume that the insurer does not recognize that asset holdings depend on $\mathbf{w}$ and f via household optimization. We believe that this is realistic because most individuals purchase private LTCI relatively late in life. Note that the demand shock, κ, is realized after LTCI is contracted.

The insurer creates a menu of contracts $\left(\pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)\right)$, $i \in \{g, b\}$ for each group of observable types that maximizes expected revenues taking into account that individual's face survival risk after insurance purchase. His maximization problem is

$$\begin{aligned}\Pi(h, \mathbf{w}, a) = \max_{(\pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a))_{i\in\{g,b\}}} & \psi\Big\{\pi^g_{f,\mathbf{w}}(a) - s_{f,\mathbf{w}}\theta^g_{f,\mathbf{w}}\big[\lambda\iota^g_{f,\mathbf{w}}(a) + kI(\iota^g_{f,\mathbf{w}}(a) > 0)\big]\Big\} \\ & + (1-\psi)\Big\{\pi^b_{f,\mathbf{w}}(a) - s_{f,\mathbf{w}}\theta^b_{f,\mathbf{w}}\big[\lambda\iota^b_{f,\mathbf{w}}(a) + kI(\iota^b_{f,\mathbf{w}}(a) > 0)\big]\Big\}\end{aligned} \quad (16)$$

subject to the incentive compatbility and participation constraints

$$(IC_i) \quad u_2(a, \theta^i_{f,\mathbf{w}}, \pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)) \geq u_2(a, \theta^i_{f,\mathbf{w}}, \pi^j_{f,\mathbf{w}}(a), \iota^j_{f,\mathbf{w}}(a)), \quad \forall i, j \in \{g, b\}, i \neq j \quad (17)$$

$$(PC_i) \quad u_2(a, \theta^i_{f,\mathbf{w}}, \pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)) \geq u_2(a, \theta^i_{f,\mathbf{w}}, 0, 0), \quad \forall i \in \{g, b\}. \quad (18)$$

Let $\tilde{h}(f, \mathbf{w}, a)$ denote the measure of agents with frailty status f, endowment $\mathbf{w}$, and asset holdings a. Then total profits for the insurer are given by

$$\Pi = \sum_{\mathbf{w}} \sum_{f} \sum_{a} \Pi(f, \mathbf{w}, a)\tilde{h}(f, \mathbf{w}, a). \quad (19)$$

4 Parametrization

Parametrizing the model proceeds in two stages. In the first stage we calibrate parameters that can be assigned to values using data without computing the model equilibrium. In the second stage we set the remaining parameters by minimizing the distance between target moments calculated using data and their model counterparts.[27] We do not formally estimate the model due to its computational intensity. To capture cross-sectional variation in income and frailty in the data we allow for 150 different income levels and 5 different frailty levels or a total of 750 risk groups. Thus, 750 distinct optimal menus need to be computed and solving for an optimal menu often requires computing several candidate solutions due to non-convexities.[28]

[27]In Section 6.5 we summarize the results from a series of robustness exercises that explore the implications of alternative parametrizations.

[28]See Section 3 and 4 of the appendix for more details on the computation and a table that summarizes the model parametrization.

4.1 Highlights of our parametrization strategy

One of our principal objectives is to assess the relative importance of choice versus no-trade in accounting for the level and pattern of LTCI take-up rates in the data. The estimated medical underwriting rejection rates that we discussed in Section 2 suggest that the extensive margin is important but they are at best a lower bound on the range of situations where there may be no basis for trade between LTC insurers and an entire risk group in the real world. These estimates only capture no-trade situations that arise as a result of information revealed during medical underwriting. However, there are many other situations where there may be no basis for trade. For instance, there may be no basis for trade between insurers and healthy, poor risk-groups who expect to qualify for means-tested NH benefits and also healthy, high- income risk-groups who prefer to self-insure because LTCI is costly to produce. Formally, to get a handle on the quantitative significance of the extensive margin one must specify the specific structure of demand and supply and that is what we do in this section.

The importance of choice versus no-trade depends on the scale of the Medicaid program, the size of administrative costs, and the extent of private information. As we explained in Section 3, absent Medicaid and administrative costs, the extensive margin does not operate and the insurer offers positive insurance to all risk groups. The scale of Medicaid is determined by the consumption floor provided to recipients and also the distribution of wealth at the point of NH entry because Medicaid benefits are means tested. We set the Medicaid NH consumption floor to the value used by Brown and Finkelstein (2008) which is based on the dollar value of transfers to Medicaid NH residents. Recall that the κ shock determines the distribution of wealth at the point of NH entry. We choose the mean of the κ shock distribution to reproduce the ratio of average wealth at NH entry to average wealth at the time of private insurance purchase, and the variance to reproduce the same ratio for quintile 5. We use the ratio of quintile 5's wealth to pin-down the variance because the extent to which higher wealth individuals have access to Medicaid is key to the relative importance of Medicaid versus supply-side frictions in accounting for the extent of private insurance. Individuals with low wealth at the time of insurance purchase are already very likely to get Medicaid benefits in the event of NH entry regardless of the size of their κ shock.

We set the administrative costs using industry-level data provided by the Society of Actuaries. The fixed cost k and variable cost parameter λ are chosen so that the model reproduces industry-level average fixed and variable costs faced by insurers.

Having fixed the scale of Medicaid and administrative costs, the next step is to parametrize the distribution of private information. We set the fraction of good types, ψ, such that the overall dispersion in private information in the model is consistent with estimates based on the data. The only direct measure of private information in HRS data is respondents' self-reported probabilities of entering a NH within the next 5 years. We set ψ such that the coefficient of variation of NH entry probabilities in the model matches the coefficient of variation of self-reported NH entry probabilities in the HRS data.[29]

[29] Ideally, we would like to use data on dispersion in self-reported NH entry risk by frailty and wealth to pin down the variation in dispersion across risk groups. However, this measure of private information is noisy, especially as sample sizes decline, and does not measure individuals' lifetime NH entry risk. For these reasons, we do not use it to parametrize $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$. Instead, in Section 5, we use this data to assess our parametrization.

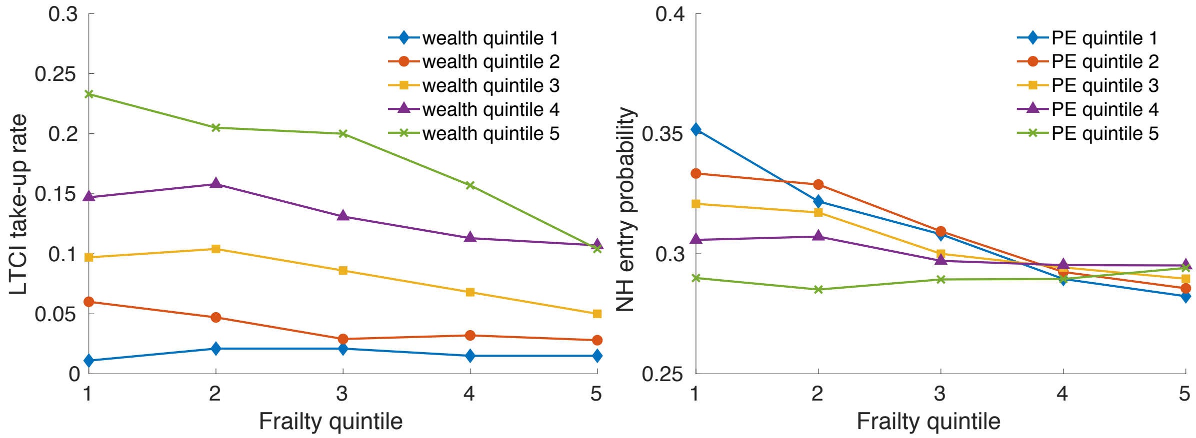


Figure 4: LTCI take-up rates by wealth and frailty quintiles (left panel) and the probability that a 65-year old will ever enter a NH by frailty and PE quintiles (right panel). LTCI take-up rates are for 62–72 year-olds in our HRS sample. NH entry probabilities are for a NH stay of at least 100 days and are based on our auxiliary simulation model which is estimated using HRS data. Frailty, wealth, and PE all increase from quintile 1 to quintile 5. The wealth quintiles reported here are marginal and not conditional on the frailty quintile, so for example only around 7% of people in frailty quintile 1 are in wealth quintile 1, while 33% are in wealth quintile 5.

The NH entry probabilities conditional on survival within each risk group, $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$, are pinned-down using data on NH entry by frailty and permanent earnings (PE) and data on LTCI take-up rates by frailty and wealth.[30] The left panel of Figure 4 shows the LTCI take-up rates of HRS respondents by frailty and wealth quintiles. LTCI take-up rates are low, 9.4% on average, decline with frailty and increase with wealth.[31] The right panel of Figure 4 shows how the lifetime NH entry probability of a 65 year-old varies across frailty and PE quintiles.[32] Notice that NH entry risk does not vary much with frailty within each PE quintile. It is essentially flat in PE quintiles 4 and 5, and decreases slightly in quintiles 1–3. Also notice that NH entry does not vary much by PE within frailty quintiles. It is slightly decreasing in PE in frailty quintiles 1–3 and there is essentially no variation in frailty quintiles 4 and 5. These patterns occur because frailty and PE are good indicators of both NH entry risk and mortality risk.

To illustrate how the model adjusts $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$ to simultaneously account for the patterns of NH entry and LTCI take-up consider PE/wealth quintiles 4 and 5. LTCI take-up rates decline with frailty in these two quintiles but the mean probability of NH entry does not

[30]We use annuitized income to proxy for PE and assign individuals the annuitized income of their household head. See Section 2 of the appendix for details.

[31]The pattern of LTCI take-up rates by frailty and wealth is robust to controlling for marital status and whether or not individuals have any children. See Section 2.4 of the appendix for details.

[32] Lifetime NH entry probabilities by frailty and PE quintile groups were obtained using an auxiliary simulation model similar to that in Hurd et al. (2013) and our HRS data. All NH entry probabilities are probabilities of experiencing a long-term (at least 100 day) NH stay. We focus on long-term NH stays because stays of less than 100 days are heavily subsidized by Medicare.

vary. The only way to generate both these patterns in the model is if the dispersion in private information, and thus the severity of the adverse selection problem, increases with frailty within these two PE/wealth quintiles. In other words, the dispersion in NH entry probabilities conditional on surviving, $\{\theta^g_{f,\mathbf{w}}, \theta^b_{f,\mathbf{w}}\}$, must go up. To provide a second example, observe that, in frailty quintiles 4 and 5, LTCI take-up rates increase with wealth but mean NH entry probabilities do not vary with PE. To account, simultaneously, for these two observations, the dispersion in $\{\theta^g_{f,\mathbf{w}}, \theta^b_{f,\mathbf{w}}\}$ must decline with PE/wealth in these frailty quintiles.

Our strategy for parametrizing ψ and $\{\theta^g_{f,\mathbf{w}}, \theta^b_{f,\mathbf{w}}\}$ allows us to determine the extent to which low LTCI take-up rates are due to choice versus no-trade. To see this, consider two alternative schemes for matching the pattern of take-up rates in the data. The first scheme is to have a large differential in NH entry between good and bad types (large θ^b to θ^g ratios within each risk group), but few bad types (a high ψ). The second scheme is to have many bad types (a low ψ), but a smaller differential in NH entry between good and bad types (small θ^b to θ^g ratios within each risk group). In our model, no-trade will play a relatively larger role in generating low take-up rates under the first scheme, while choice menus will play a relatively larger role under the second. No-trade menus will play a larger role under the first scheme because the large differential between θ^b and θ^g makes cross-subsidizing menus unprofitable but large θ^b also makes choice menus unprofitable. Choice menus will play a larger role under the second scheme because the large fraction of bad types makes cross-subsidizing menus unprofitable, but lower θ^b means the insurer can still make profits by insuring bad types on their own. Thus, choice menus will still be profitable.

Consistently, in Section 6.5, we document that lowering ψ and then reparametrizing $\{\theta^g_{f,\mathbf{w}}, \theta^b_{f,\mathbf{w}}\}$ to match the LTCI take-up rates results in a higher fraction of choice menus. However, this second scheme also produces too little overall dispersion in private information. In practice, the reduction in dispersion due to reducing the ratios of θ^b to θ^g within risk groups dominates the increase in dispersion due to reducing ψ. Thus, reproducing the overall dispersion in private information in the data identifies the relative role of optimal menus featuring choice versus no-trade in the model.

4.2 Functional forms and first stage calibration

We assume constant-relative-risk-aversion utility such that

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

Individuals cover a substantial fraction of NH expenses using their own resources. Given the size of these expenses, it makes sense to assume that households are risk averse and thus willing to pay a premium to avoid this risk. A common choice of the risk aversion coefficient in the macroeconomics incomplete markets literature is $\sigma = 2$. We use this value.

The distribution of frailty in the model is calibrated to replicate the distribution of frailty of individuals aged 62–72 in our HRS sample. We focus on 62–72 year-old individuals because frailty is observed by the insurer at the time of LTCI purchase. In our HRS sample, the frailty of 62–72 year-old individuals is negatively correlated with their PE. To capture this feature of the data we assume that the joint distribution of frailty and the endowment stream,

Table 1: Mean frailty by PE quintile in the data and the model.

	PE Quintile 1	2	3	4	5
Data	0.23	0.22	0.19	0.16	0.15
Model	0.23	0.20	0.19	0.17	0.15

Data source: Authors' calculations using our HRS sample.

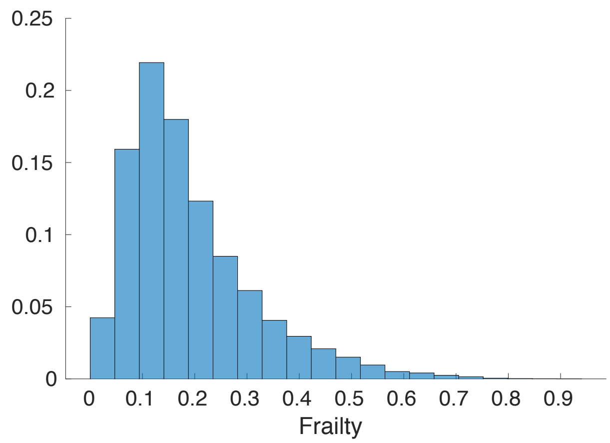


Figure 5: Distribution of frailty for 62–72 year-olds in our HRS sample. Severity of frailty is increasing with the index value and the maximum is normalized to one.

$h(f, \mathbf{w})$, is a Gaussian copula. This distribution has two attractive features: the marginal distributions do not need to be Gaussian and the dependence between the two marginal distributions can be summarized by a single parameter $\rho_{f,\mathbf{w}}$. The value of this parameter is set to -0.29 so that the variation in mean frailty by PE quintile in the model is as observed in the data. Table 1 shows the data values and model counterparts.

Figure 5 shows the empirical frailty distribution. We approximate it using a beta distribution with $a = 1.54$ and $b = 6.30$. The parameters of the distribution are chosen such that mean frailty in the model is 0.19 and the Gini coefficient of the frailty distribution is 0.34, consistent with their counterparts in the data. When computing the model, we discretize frailty into a 5-point grid. We use the mean frailty of each quintile of the distribution as grid values.

The marginal distribution of endowments is assumed to be log-normal. We equate endowments to the young with permanent earnings and normalize the mean young endowment to 1. This is equivalent to a mean permanent earnings of $1,049,461 in year 2000 which is approximated as average earnings per adult aged 18–64 in year 2000 multiplied by 40 years.[33] The standard deviation of the log of endowments to the young is set to 0.8 because it implies that the Gini coefficient for the young endowment distribution is 0.43. This value

[33]To derive average earnings per adult aged 18-64 in year 2000 we divide aggregate wages in 2000 taken from the Social Security Administration by number of adults aged 18-64 in 2000 taken from the U.S. Census.

is consistent with the Gini coefficient of the permanent earnings distribution for individuals 65 and older in our HRS sample.

Endowments to the old are a stand in for retirement income which is comprised primarily of income from social security and private pension benefits. We assume that the income replacement ratio (retirement income relative to pre-retirement income) is linear in logs. Purcell (2012) calculates income replacement ratios for HRS respondents. Using his calculations, we set the level and slope of the replacement rate function such that the median replacement rate of retirees in the bottom pre-retirement income quartile is 64% and the median rate for retirees in the top quartile is 50%.[34] The resulting average replacement rate in the baseline economy is 57%.

The consumption demand shock, κ, captures the uncertainty individuals face at the time of LTCI purchase about their resources later in life when a NH event may occur. This uncertainty is, in part, due to uncertainty about the date of NH entry itself. Since the distribution of NH entry ages is left-skewed, we assume that the distribution of the κ shock, $q(\kappa)$, is also left-skewed.[35] This is achieved by setting $q(\kappa)$ such that $1-\kappa$ has a truncated log-normal distribution over $[0.2, 0.8]$.[36] The mean and variance of κ, μ_κ and σ_κ^2, are determined in the second stage.

We estimate the risk of a long-term stay in a NH using HRS data and the questions in that survey do not distinguish between stays in skilled nursing facilities (SNF) and stays in assisted living communities or residential care centers (RCC). Thus, when estimating the average cost of a NH stay we take a weighted average of SNF and RCC expenses. In practice residential LTC expenses have two components. The first component is nursing and medical care and the second component is room and board. We interpret the room and board component as being part of consumption and thus a choice and not an expense shock. Using data from a variety of sources, we estimate that the average medical and nursing expense component of residential LTC costs was \$32,844 per annum in 2000 and the average benefit period was 2.976 years. Multiplying the annual medical and nursing cost by the average benefit period yields total medical expenses of \$97,743 or a value of m of 0.0931 when scaled by mean permanent earnings.[37]

We set the consumption floor provided by Medicaid, $\underline{c}_{NH}$, and the consumption floor for those who do not enter a NH, $\underline{c}_o$, to the same value: \$6,540 a year. As mentioned above, this value is taken from Brown and Finkelstein (2008) and consists of a consumption allowance of \$30 per month and housing and food expenses of \$515 per month. The former number is based on Medicaid administrative rules and the latter figure was the monthly amount that SSI paid a single elderly individual in 2000. We assume that the third period of the model has the same length as the average duration of NH entry conditional on a long-term stay. Thus, we multiply the annual consumption floor by 2.976 years to come up with the total size of the consumption floor. The resulting value of c_{nh} is 1.855% of mean permanent earnings.

Having calibrated the joint distribution of frailty and the endowment stream, $h(f, \mathbf{w})$, we

[34] These estimates are the median replacement rates of retirees who have been retired for at least 6 years. See Purcell (2012), Table 4.

[35] Murtaugh et al. (1997) estimate the distribution of NH entry ages.

[36] The baseline parametrization is robust to expanding the range of κ values within $[0, 1]$.

[37] See Section 4 of the appendix for details and data sources.

use it to assign individuals in the model to frailty and PE quintiles, and thereby partition the population into 25 groups, one for each frailty/PE quintile combination. To reduce the number of parameters, we assume that individuals within the same group have the same survival probability $s_{f,\mathbf{w}}$ and the same set of NH entry probabilities $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$.[38]

The 25 survival probabilities are set to the probability that a 65 year-old will survive to either age 80 or until a NH event occurs.[39] We use survival until age 80 *or* a NH event because this way, regardless of which one we target, our parametrized model will match both the unconditional NH entry probabilities reported in Figure 4 and NH entry probabilities conditional on survival which we report in Section 4 of the appendix. The resulting survival probabilities of each frailty and PE quintile are also reported in the appendix. Not surprisingly, the relationship between frailty and survival is negative in all PE quintiles.

Finally, the risk-free real return, r, is not separately identified from the preference discount factor β. We normalize it to 0% per annum.[40]

4.3 Second stage: simulated moment matching strategy

The set of parameters left to pin down are the preference discount factors (β, α), the consumption shock distribution parameters $(\mu_\kappa, \sigma_\kappa)$, the administrative cost parameters (λ, k), the fraction of good types ψ, and the 25 NH entry probability pairs: $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$ for each frailty/PE quintile combination. These parameters are chosen to minimize the distance between equilibrium moments of the model and their data counterparts. Even though all of these parameters are chosen simultaneously through the minimization procedure, each parameter has a specific targeted moment.

The preference discount factor ,β, in conjunction with the interest rate and σ determines how much people save for retirement. It is chosen such that the model reproduces the average wealth of 62–72 year olds in our HRS sample relative to average lifetime earnings. This value is 0.222 in the data and 0.229 in the model. The resulting annualized value of β is 0.94.[41]

On average individuals in our dataset enter a NH at age 83 or about 18 years after they retire. The parameter α captures the discounting between the age of retirement and LTCI purchase, and the age when a NH event is likely to occur. The more that individuals discount the NH entry period, the larger the fraction of NH residents who will be on Medicaid. Thus our choice of α targets the Medicaid recipiency rate of NH residents in our HRS sample. The target rate is 46%, the model rate is 48%, and the value of α is 0.20.[42]

We set the consumption shock distribution parameters, $(\mu_\kappa, \sigma_\kappa)$, to target two data facts. The first data target is the average wealth of NH entrants immediately before entering the

[38] We wish to emphasize that these groups are not risk groups because individuals in a given group are not identical to the insurer. The insurer observes 150 distinct levels of permanent earnings and thus will offer different menus to individuals in a given group.

[39] Survival probabilities by frailty and PE quintiles are estimated using HRS data and our auxiliary simulation model. See footnote 32.

[40] This normalization only impacts the value of β and for our analysis, which does not involve any welfare calculations, is innocuous.

[41] Our choice of this age group is based on two considerations. First, if we limit attention to those aged 65 we would only have a small number of observations. Second, the average age when individuals purchase LTCI in our sample is 67 and this is the midpoint of the interval we have chosen.

[42] Our Medicaid recipiency rate target is lower than other estimates. But, this reflects the fact that in

Table 2: LTCI take-up rates by wealth and frailty: data and model

	Data			Model		
Frailty	Wealth Quintile			Wealth Quintile		
Quintile	1–3	4	5	1–3	4	5
1	0.071	0.147	0.233	0.073	0.145	0.245
2	0.065	0.158	0.205	0.069	0.165	0.202
3	0.049	0.131	0.200	0.048	0.128	0.245
4	0.037	0.113	0.157	0.032	0.122	0.151
5	0.025	0.107	0.104	0.029	0.102	0.118

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. We merge wealth quintiles 1–3 because take-up rates are very low for these individuals. Data source: 62–72 year olds in our HRS sample.

NH relative to the average wealth of 62–72 year olds. This ratio is 0.62 in our dataset and 0.68 in our model.[43] The second data target is the ratio of average wealth in quintile 5 of NH entrants immediately before entering the NH relative to the average wealth in quintile 5 at age 62–72. The ratio is 0.70 in our dataset and 0.66 in the model. The resulting mean and standard deviation of the the distribution of κ are respectively 0.60 and 0.071. So on average individuals lose 60% of their wealth between the time they purchase LTCI and the time they enter the NH.

As discussed in Section 2, LTCI insurers incur large administrative costs because they conduct extensive medical underwriting and pay large commissions to the brokers who sell their products. We divide administrative costs into a fixed and variable cost component. Eaton (2016) reports that fixed administrative costs, which include underwriting costs and costs of paying claims, were 20% of present-value premium on average in 2000. Variable costs consist of commissions paid to agents and brokers. They amounted to 12.6% of present-value premium on average in 2000. We choose k and λ to reproduce these targets. The resulting values of k and λ are 0.019 and 1.195, respectively.

The coefficient of variation of self-reported 5-year NH entry probabilities is 0.94 in the HRS data.[44] We choose the fraction of good types, ψ, such that the coefficient of variation of NH entry probabilities in the Baseline economy replicates this value. The resulting value of ψ is 0.709.

As we explained above, we assume that individuals within the same frailty and PE

our HRS sample a NH stay includes a stay in an RCC and Medicaid take-up rates are much lower in RCC facilities. For instance, data from the CDC national survey of LTC providers (see Harris-Kojetin et al. (2016)) reports that 63% of individuals in skilled nursing facilities receive Medicaid benefits but only 15% of individuals in RCC facilities receive Medicaid benefits. According to Spillman and Black (2015), 36% of NH residents are in RCC facilities. These numbers imply a similar Medicaid take-up rate of 48%.

[43] To calculate this number in the data, we average the wealth of NH entrants in the wave that precedes their NH entry wave.

[44] This is the value when we do not count reports of 0, 50% or 100%. Including these additional observations in any combination weakly increases the coefficient of variation. In Section 6.5, we discuss the robustness of our results to this choice of data target.

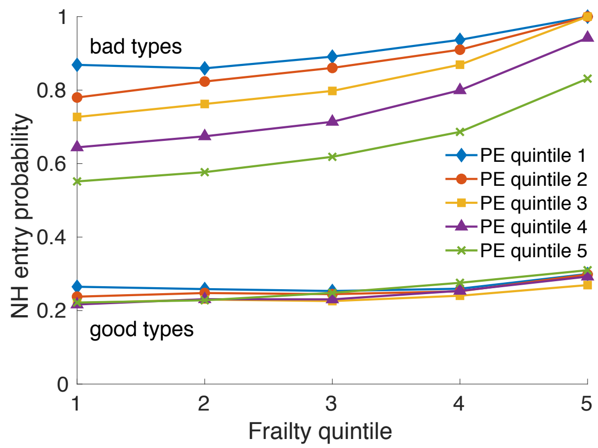


Figure 6: Nursing home entry probabilities conditional on surviving for good and bad types by frailty and PE quintile in the Baseline economy.

quintiles have the same set of NH entry probabilities, $\{\theta^b_{f,\mathbf{w}}, \theta^g_{f,\mathbf{w}}\}$. We pin down these 25 NH entry probability pairs using two sets of targets. The first set of targets are the 25 probabilities of entering a NH for a lifetime stay by frailty/PE quintile combination reported in the right panel of Figure 4. By targeting these probabilities we are ensuring that the average NH entry probability in each frailty/PE quintile group replicates its estimated value based on the HRS data. The second set of targets are the 15 LTCI take-up rates of individuals in all combinations of quintiles 1–3, 4, and 5 of the wealth distribution and quintiles 1 through 5 of the frailty distribution reported in the lower panel of Table 2. In order to identify these 50 parameters using only 40 moments, we assume that the ratio of NH entry probabilities within a risk group is constant across wealth quintiles 1–3 within each frailty quintile.[45] Our decision to restrict the parameters in this way is based on two considerations. First, recall from Figure 4 that only a very small number of individuals in quintiles 1 and 2 have LTCI in our dataset. Second, in the model, no individuals in these quintiles buy LTCI because they are guaranteed to get Medicaid if they incur a NH event.[46] The resulting NH entry probability pairs are displayed in Figure 6. Observe that the dispersion in the $\theta's$ increases with frailty but declines with PE. From this we see that the model is indeed assigning a bigger role to private information in frail and poor risk groups as we suggested in Section 4.1.

Table 2 reports the 15 LTCI take-up rates in the Baseline economy. The fit of the model is not perfect due to the fact that we discretize the state space to compute the model. Note, however, that the take-up rates generated by the model increase with wealth and decline with frailty for both the rich and poor. The model also does a good job of reproducing the

[45]Specifically, we assume that $\theta^b_{f,\mathbf{w}}/\theta^g_{f,\mathbf{w}}$ is constant across wealth quintiles 1–3 within each frailty quintile. This produces 10 restrictions such that, together with the 40 other moments, the 50 parameters are exactly identified.

[46]This difference between the model and the data is present for a variety of reasons including measurement error, our parsimonious specification of the Medicaid transfer function, and the fact that we have not modeled all shocks faced by retirees such as spousal death.

Table 3: Standard deviation of self-reported (private) NH entry probabilities by frailty and permanent-earnings quintiles: data and model

	Frailty Quintile				
	1	2	3	4	5
Data	1.00	1.00	1.03	1.27	1.47
Model	1.00	1.08	1.20	1.31	1.47
	Permanent Earnings Quintile				
	1	2	3	4	5
Data	1.00	0.92	0.85	0.79	0.76
Model	1.00	0.96	0.91	0.78	0.59

The standard deviations (SDs) are normalized such that the SD of frailty quintile 1 is 1. Data values are SDs of self-reported probabilities of entering a NH in the next 5 years for individuals aged 65–72 excluding observations where the probability is 0, 100% or 50%. The pattern in the data is robust to variations in the way we construct the SDs including how we handle those reporting a probability of 0, 100% or 50%. Data source: Authors' calculations using our HRS sample.

average LTCI take-up rate. In our HRS sample, 9.4% of retirees aged 62–72 have LTCI and in the model 9.7% of 65 year-olds have a nonzero LTCI contract. The fact that we are able to reproduce the average LTCI take-up rate suggests that the restrictions we have imposed on the $\theta^b_{f,\mathbf{w}}$'s for wealth quintiles 1-3 are broadly consistent with our data.

5 Assessing the model parametrization

Dispersion of private information by frailty and permanent earnings. One way to assess this aspect of our model is to provide independent evidence that dispersion in private NH entry probabilities, and thus the severity of the private information friction, increases with frailty and decreases with PE. The first and third rows of Table 3 report normalized standard deviations of self-reported NH entry probabilities for 65–72 year-old HRS respondents by frailty and PE quintile. These probabilities are not exactly comparable to the private NH entry probabilities in the model for two reasons. First, they are self-reported probabilities of NH entry in the next 5 years whereas the model values are lifetime NH entry probabilities. Second, the self-reported probabilities are noisy and in some instances sensitive to how one cleans the data. For instance, 1/3 of respondents report 0.5 and another third report either 0 or 1 in the raw data. We choose to omit these responses. The second and fourth rows of the table report the distribution of private NH entry probabilities by frailty and PE quintile that emerge from the model. Despite the noise, the dispersion of private information is increasing in frailty and decreasing in PE in both the data and the model. This pattern of dispersion is consistent with Hendren (2013)'s findings, discussed in Section 2.1, that adverse selection is more severe among individuals that are more likely to rejected by LTC insurers. Notice that the model also does a good job in matching the extent of variation in dispersion across both frailty and PE quintiles.

Table 4: Distribution of insurance across NH residents: data and model

	LTCI	Medicaid	Both	Neither
Data	8.2	45.6	2.7	43.4
Model	9.5	47.6	0.3	42.6

Percent of NH residents covered by LTCI only, Medicaid only, both, or neither in the data and the model. Data source: Authors' calculations using our HRS sample.

Distribution of insurance. Table 4 shows the distribution of insurance across NH residents in the model and the HRS data. None of these moments were explicitly targeted when parametrizing the model. Yet, the fit between the model and data is very good. The model even predicts that some NH residents receive both private LTCI and Medicaid benefits. In the model, these are individuals who, ex-ante, bought LTCI because they would not be covered by Medicaid for all realizations of the demand shock but, ex-post, drew a realization of κ that resulted in Medicaid eligibility.

Pricing and coverage of LTCI. One of our objectives is to propose a model of an insurer's optimal contracting problem that is quantitatively relevant. Thus, the model's implications for pricing and coverage levels of insured individuals are particularly important. Since pricing and coverage statistics where not targeted when parametrizing the model, they are a clean way to assess the model's performance.

Average pricing and coverage levels of LTCI products in our model are consistent with observations from the U.S. LTCI market. Recall that Brown and Finkelstein (2007) and Brown and Finkelstein (2011) find that the average load in the LTCI market is in the range 0.18 and 0.5, depending on whether or not the loads are adjusted for policy lapses and the sample period. The average load in our model at 0.41 falls in middle of this range. In Section 2 we explained that typical coverage levels for LTCI products range between one-third and two-thirds of expected lifetime NH expenses. Insurance contracts in our model offer indemnities that cover on average 58% of NH medical costs.

Brown and Finkelstein (2007) also find that the relationship between loads and comprehensiveness is non-monotonic and that for some individuals loads are negative. Table 5, shows that average loads and coverage don't vary systematically with wealth. However, average loads are increasing in frailty and coverage levels are declining in frailty. Thus, frail individuals pay more for LTCI and receive less coverage according to the model.

Table 5 also reports coverage and loads by private information type. It is immediately clear from these results that one way the insurer responds to adverse selection is to offer bad-risk types in insured risk groups more coverage at a lower price. In virtually all wealth and frailty quintiles, bad types have negative loads indicating that they are getting a good deal relative to the actuarially fair benchmark. Good types, in contrast, have large and positive loads at all wealth and frailty quintiles. The combination of negative loads for bad types and positive loads for good types highlights the fact that the optimal contracts feature cross-subsidization. Revenues from good types are used by the insurer to subsidize contracts

Table 5: Comprehensiveness and individual loads by private type and frailty and wealth quintiles in the Baseline economy.

	Wealth Quintile				
	1	2	3	4	5
Average					
Fraction of NH costs covered	NA	NA	0.552	0.607	0.581
Load	NA	NA	0.408	0.389	0.406
Good risks (θ^g)					
Fraction of NH costs covered	NA	NA	0.507	0.507	0.514
Load	NA	NA	0.631	0.605	0.558
Bad risks (θ^b)					
Fraction of NH costs covered	NA	NA	0.711	0.711	0.816
Load	NA	NA	-0.082	-0.46	0.056
	Frailty Quintile				
	1	2	3	4	5
Average					
Fraction of NH costs covered	0.578	0.592	0.578	0.572	0.564
Load	0.400	0.394	0.405	0.409	0.414
Good risks (θ^g)					
Fraction of NH costs covered	0.514	0.517	0.518	0.492	0.487
Average load	0.581	0.589	0.591	0.607	0.620
Bad risks (θ^b)					
Fraction of NH costs covered	0.763	0.753	0.774	0.739	0.736
Load	-0.004	-0.005	-0.017	-0.020	-0.031

The fraction of NH costs covered is the average indemnity divided by the medical and nursing expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance. NA denotes cases where LTCI take-up rates are zero.

to bad types within a given risk group.

In the model, the insurer is free to create a separate menu for each risk group and, in equilibrium, offers hundreds of risk-group-specific menus. Table 5 indicates that, on the one hand, these menus feature very different contracts for good and bad private information types. On the other hand, the optimal contracts are quite similar across alternative wealth and frailty levels for a given private information type. For instance, coverage levels and loads for good types only vary by about 7 percentage points across wealth quintiles. It is consequently conceivable that modeling a small fixed cost for writing each distinct menu could result in a much smaller set of menus. We do not pursue this strategy here because introducing this type of fixed cost significantly complicates the insurer's problem.[47] Still, the results in Table 5 suggest that the incremental return associated with offering a custom

[47]Finding the optimal set of menus with fixed menu costs is a challenging combinatorics problem because there are a very large number of risk groups and thus combinations of menus that have to be considered. For each posited set of menus one has to verify that each risk group's incentive-compatibility and participation constraints hold.

menu to each risk group could be small.

6 Results

6.1 LTCI take-up rates and rejections

The model has two different ways to account for low LTCI take-up rates. One way is to offer menus that feature a choice to a range of risk groups. That is to offer risk groups menus that consist of two contracts, a non-zero contract and a $(0, 0)$ contract. As we explained above, in this situation good-risk types choose the no-insurance option. The other way to produce low LTCI take-up rates is via screening along the extensive margin — to offer some risk groups no insurance at all. It turns out that only 0.11% of individuals choose not to purchase LTCI. Thus, rejections are the main mechanism the model uses to generate low LTCI take-up rates. Consistently, as the first column of Table 6 shows, the rejection rate in our Baseline economy is 90.1%. It is 100% for individuals in PE quintiles 1 and 2 and declines with permanent earnings in quintiles 3–5. However, the rejection rates are non-monotonic among the highest PE individuals. Rejection rates are only 58.8% among individuals in the top 5% of PE but then rise to 100% for those in the top 1%. Individuals with the highest PE prefer to self-insure NH risk.

The pattern of rejections reported in the first column of Table 6 are consistent with the estimates of rejection rates that we reported in Section 2. Rejection rates decline with wealth in both the model and the data. Rejection rates in the model are much larger than the empirical rejection rates which ranged from 36% to 56% for 55–66 year old HRS respondents. Recall that the empirical rejection rates are best interpreted as providing a lower bound on situations where no trade might occur. They only capture no trade that arises because, from the insurer's perspective, insuring a particular risk group is not profitable. They do not capture no-trade that arises because individuals in some risk groups perceive LTCI to be too expensive. However, survey results in Ameriks et al. (2016) suggest that the high cost of LTCI is also an important reason for low take-up rates.

6.2 Insurance ownership and NH entry

Our finding on the quantitative significance of no-trade menus raises the possibility that empirical tests for adverse selection based on estimated correlations between insurance ownership and loss occurrence may have weak power. These tests are based on the standard theory of adverse selection with a single source of private information which predicts that, if adverse selection is present in the market, LTCI holders should have higher NH entry rates than non-holders (see Chiappori and Salanie (2000)). As we discussed in Section 2.1, Finkelstein and McGarry (2006) use them to test for adverse selection in the U.S. LTCI market. They find that LTCI holders, if anything, have lower NH entry rates than non-holders and that this is the case despite documenting direct evidence that individuals have private information about their NH risk and that they act on this risk by purchasing LTCI.

Finkelstein and McGarry (2006) conclude that, to reconcile their conflicting set of findings, there must be multiple sources of private information present in the U.S. LTCI market.

Table 6: Rejection rates in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information Economies

Scenario Description	Baseline	No Admin. Costs $\lambda = 1,\ k = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information $\theta^i_{f,\mathbf{w}}$ public
Average	90.1	38.7	9.4	62.5
By PE Quintile				
1	100	100	27.4	100
2	100	93.4	0.0	99.6
3	85.7	0.0	0.0	54.1
4	83.9	0.0	0.0	29.1
5	81.2	0.0	19.8	29.7
High PE				
top 10	75.1	0.0	39.5	30.4
top 5	58.8	0.0	76.2	31.7
top 1	100	0.0	100	100
By receiving Medicaid NH benefits conditional on surviving				
Would	47.6	37.0	5.5	43.9
Would not	42.5	1.6	4.0	18.6

Rejection rates are percentage of individuals who are only offered a single contract of $(0, 0)$ by the insurer. Note that, in the first nine rows, the figures are the percentage of individuals in that group. However, the bottom two rows of the table are a decomposition of the average rejection rate for that economy.

However, our model with a single source of private information and an active extensive contracting margin is able to generate each of their empirical results. First, Finkelstein and McGarry (2006) find a positive correlation between self-assessed NH entry risk and NH entry, even after controlling for observable health, and interpret this as evidence of private information. We have explicitly modeled private information and adverse selection and, as Figure 6 shows, our model delivers this correlation by construction. Second, they find that individuals act on their private information by documenting a positive correlation between self-assessed NH entry risk and LTCI ownership. The baseline economy has this property too. The LTCI ownership rate of bad types is 10.6% while the ownership rate of good types is 10.2%. Moreover, bad types have a higher LTCI ownership rate than good types no matter whether or how we control for the information set of the insurer, or whether or not we condition on survival. Third, the correlations between LTCI ownership and NH entry rates in the Baseline economy are small and can be negative. Table 7 reports NH entry rates conditional on survival of LTCI holders and non-holders. Only 36.9% of LTCI holders in

[48] We believe that the NH entry rates conditional on survival are the most comparable to Finkelstein and McGarry (2006)'s findings given that they only look at NH entry within 5 years of observed LTCI ownership. That said, with no controls, our model still generates a negative, albeit smaller, correlation even if we do not condition on survival. The reason conditioning on survival matters is because it impacts the correlation between average NH entry and LTCI take-up rates across risk groups (see Figure 4).

Table 7: NH entry rates of LTCI holders and non-holders in the Baseline economy

		Frailty Quintile				
	Average	1	2	3	4	5
LTCI holders	36.9	33.4	36.0	37.2	41.2	47.5
Non-holders	40.7	35.9	37.9	40.1	43.0	49.1

Numbers are percent of survivors to the very old stage of life who enter a NH.

the Baseline economy enter a NH whereas 40.7% of non-holders enter, consistent with the negative correlation Finkelstein and McGarry (2006) find when they do not control for the insurer's information set.[48]

Chiappori and Salanie (2000) ascertain that to properly test for the presence of private information one must fully control for the information set of the insurer. Finkelstein and McGarry (2006) consider two different sets of controls. The first set only controls for observable variation in health. The second set controls for both observable health variables and individuals' wealth and income quartiles. In both cases, they find a small negative but not statistically significant correlation. Only when they consider a special sample of individuals who are in the fourth quartile of the wealth and income distributions and have no health issues that would likely lead them to be rejected by insurers do they find a statistically significant negative correlation.

Consistent with their findings, as Table 7 shows, if we only control for frailty, we continue to find a negative correlation but the size of the differential between the entry rates of non-holders and holders is now smaller in each group.[49,50] If, in addition to frailty, we also control for wealth and income quartile, the differences in the entry rates between non-holders and holders becomes even smaller.[51] In addition, the correlation is negative for precisely half of the groups and positive for the other half, and the average differential is essentially zero.[52] Finally, like Finkelstein and McGarry (2006), if we focus on individuals in the top wealth and income quartile and the lowest frailty quintile, we find a negative correlation between LTCI ownership rates and NH entry. The NH entry rate of LTCI holders in this group is 31.8% while the entry rate of non-holders is 32.2%.[53]

The intuition for our findings is as follows. First, to understand how the model produces

[49] We check for conditional independence of NH entry and LTCI ownership because Chiappori and Salanie (2000) point out that this strategy, which is the basis of their χ^2 statistic, is more robust to nonlinearities.

[50] If we do not condition on survival, some of the differentials in the upper frailty quintiles flip sign but the absolute value of the difference in NH entry rates between holders and non-holders becomes even smaller.

[51] These results are not reported in the table because the number of groups is so large.

[52] Equally weighting each group, the average NH entry rate of LTCI holders is 0.07 percentage points higher than the entry rate of non-holders.

[53] One difference between the moments generated by our model and the statistics reported in Finkelstein and McGarry (2006) is that they compute correlations between LTCI ownership and NH entry within the 5 years after observing ownership. NH entry rates in our model are the lifetime rates. Alternatively, one could construct an empirical measure of lifetime NH entry risk to compare to our model results. However, this is not straightforward because lifetime NH risk of HRS respondents is not directly observable and would have to be estimated using an auxiliary model. This creates an additional source of noise and specification error. In our view it is best to compare our model results with the empirical findings of Finkelstein and McGarry (2006).

small positive correlations between LTCI ownership and NH entry it is useful to return to Figure 1 which shows the various types of optimal menus that can occur. Observe that only one of these types, the one displayed in Figure 1e, will generate a non-zero (positive) correlation between LTCI ownership and NH entry within a risk group. This menu features no insurance for good risks and positive insurance for bad risks. Under all the other optimal menus the correlation is zero because either both risk-types are insured or neither risk-type is insured. In other words, only optimal menus of the type illustrated in Figure 1e will provide identification of adverse selection using empirical tests that rely on correlations between LTCI ownership and NH entry. Now recall that only 0.11% of individuals are offered this type of menu in the Baseline economy. The fact that this type of menu is so infrequent means that these empirical tests of adverse selection will have low power.

Second, to understand how the model produces negative correlations between LTCI ownership and NH entry recall that adverse selection is more pronounced in poor and frail risk groups and, consequently, rejection rates, like NH entry rates, decrease in permanent earnings and increase in frailty. These facts create the possibility of finding a negative correlation if the information set of the insurer and econometrician are different such that the econometrician bunches two or more risk groups together. In this scenario, the negative correlation between LTCI ownership and NH entry across risk groups may dominate the positive correlations within risk groups. Given that very few risk groups in the Baseline economy feature a non-zero positive correlation, it is not surprising that when risk groups are bunched together a negative correlation is found.

We wish to point out that a more powerful way to test for adverse selection in our model would be to look at the correlation between NH entry and the comprehensiveness of LTCI coverage. All optimal menus with positive amounts of insurance have the property that bad risk types have more coverage than good risk types. Unfortunately, the HRS data, which is the data used by both Finkelstein and McGarry (2006) and us, only has information on LTCI ownership, not on the comprehensiveness of coverage. Note that, even if data on comprehensiveness was available, the bunching effect would still be operative.

6.3 Role of demand- and supply-side frictions

We showed in Section 3 that, at a qualitative level, low take-up rates, rejections, and partial coverage could be accounted for, independently, by either supply-side frictions (administrative costs and adverse selection) or demand-side frictions (Medicaid). We would like to understand the significance of each of these mechanisms for our results. To help distinguish between them, we will compare the Baseline economy with three other economies. In each economy, endowments and the interest rate are held fixed at their baseline values. In the No Administrative Costs economy, we remove the insurer's variable and fixed costs by setting $\lambda = 1$ and $k = 0$. In the No Medicaid economy, the NH consumption floor $\underline{c}_{NH}$ is reduced to 0.001.[54] Finally, in the Full Information economy, which is designed to understand the effects of private information, the insurer can directly observe each individual's true NH risk exposure, $\theta^i_{f,\mathbf{w}}$.

[54]We do not reduce $\underline{c}_{NH}$ to zero because then some individuals would experience negative consumption. Also note that the non-NH consumption floor, $\underline{c}_o$, does not vary across economies.

Table 8: LTCI take-up rates by wealth and frailty: Baseline and Full Information economies

	Baseline			Full Information		
Frailty	Wealth Quintiles			Wealth Quintiles		
Quintile	3	4	5	3	4	5
1	0.183	0.145	0.245	0.565	0.709	0.694
2	0.175	0.165	0.202	0.512	0.709	0.709
3	0.142	0.128	0.245	0.418	0.709	0.708
4	0.111	0.122	0.151	0.409	0.709	0.711
5	0.111	0.102	0.118	0.413	0.709	0.699

The LTCI take-up rates in wealth quintiles 1 and 2 are zero in both economies.

Column 2 of Table 6 reports rejection rates in the No Administrative Costs economy. When administrative costs are absent, the extensive margin is no longer used as a screening device for risk groups consisting of more affluent individuals. The average rejection rate drops from 90.1% to 38.7% and all individuals in PE quintiles 3–5 purchase LTCI. However, rejection rates in the two lowest PE quintiles are still very high.

Medicaid, in contrast, is of central importance in accounting for low LTCI take-up rates among the poor as shown in column 3 of Table 6. Rejections decline sharply in PE quintiles 1 and 2 when Medicaid is removed. Rejection rates are still positive in PE quintile 1 because some individuals in that quintile are so poor that they cannot afford NH care and must rely on Medicaid even though the Medicaid consumption floor is extremely low. Interestingly, Medicaid also reduces rejection rates among higher PE individuals. Rejection rates fall in PE quintiles 3-5 and also in the top decile. The reason that rejection rates fall in these more affluent groups is because even relatively high PE individuals will satisfy the means-test threshold for Medicaid in some states of nature. Our result is consistent with previous findings by Braun et al. (2015) and De Nardi et al. (2013) who find that even high PE individuals value Medicaid. Finally, observe that rejection rates actually increase in the top 5% PE group. Removing Medicaid increases saving and thus wealth at the time that individuals contract for LTCI. For individuals in the top 5% PE group, this effect is very pronounced. They have more wealth and thus are in a better position to self-insure.

Finally, consider the role of private information by comparing column 1 with the final column of Table 6. Absent private information, rejection rates fall from 90.1% to 62.5%.[55] This decline is primarily due to a decline in rejection rates of more affluent individuals. Individuals in PE quintiles 3–5 all experience declines in rejections and rejections also fall in the top 10% and top 5% PE groups. Removing private information increases profitability for the insurer because he can now price discriminate on the basis of true-risk exposure. This has a larger effect on rejection rates of higher income individuals because the option value of Medicaid is relatively smaller for them.

Administrative costs and private information have similar effects in that they both pri-

[55]Note that choice menus do not exist in the Full Information economy because each risk group is only offered one contract. Thus the rejection rate in this economy is one minus the LTCI take-up rate.

marily impact the rejection rates of higher PE individuals. This raises the following question. Is private information essential to generate the extent and pattern of rejections, and hence LTCI take-up, observed in the data or could the model do just as well if we abstracted from it? Table 8 reports LTCI take-up rates in the Baseline and Full Information economies. The table shows that, not only does the presence of private information reduce the extent of LTCI take-up, but it also plays an important and unique role in allowing the model to account for the empirical pattern of LTCI take-up among affluent individuals. Notice that, in wealth quintiles 4 and 5 of the Full Information economy, the LTCI take-up rates exhibit the wrong pattern by frailty (see also Table 2). LTCI take-up rates in these two wealth quintiles are declining in frailty in the data and the Baseline economy. However, in the Full Information economy, they are constant in frailty in wealth quintile 4 and hump-shaped in frailty in quintile 5. As we discuss in Section 6.5, even if we reparametrize the Full Information economy, it is unable to match both the level and pattern of LTCI take-up in the data. These findings show that the extent and pattern of private information in the market documented by Finkelstein and McGarry (2006) and Hendren (2013) is an important driver of low LTCI take-up rates that decline with frailty.

In the Baseline economy, like in the U.S., a substantial fraction of NH costs are paid for OOP. There are two reasons for this. First, as we explained in Section 4, very few NH residents have both LTCI and Medicaid, and LTCI contracts only provide partial coverage. Second, many individuals who do not purchase LTCI are too affluent to quality for Medicaid NH benefits and have no recourse but to pay for their NH care OOP. The bottom two rows of Table 6 report statistics related to this second source of OOP payments. In these two rows, individuals in the model who are rejected by the insurer are divided into two groups: those who, if they survive to the very old age and enter a NH, would qualify for Medicaid NH benefits and those who would not. In the baseline economy, 42.5% of rejected individuals would, if they enter a NH, be completely uninsured because they would be too affluent to satisfy the Medicaid means test. This group pays all of their NH expenses OOP. Notice that the insurance coverage gap decreases substantially if either of the two supply-side distortions is absent. In the No Administrative Cost economy, only 1.6% of those who are rejected would end up paying for all their care OOP and, in the Full Information economy, only 18.6% of individuals would find themselves in this situation. The reason the coverage gap is so small in these two economies is because rejection rates are much lower among high PE individuals. Thus most high PE individuals are covered by private LTCI, while less affluent individuals continue to receive extensive Medicaid coverage. These findings indicate that reductions in the extent of supply-side distortions in the U.S. LTCI market could lead to large reductions in the fraction of individuals paying OOP for NH care.

6.4 Coverage, loads and profits

Taken together the results in Sections 5 and 6.1 show that the insurer is screening risk groups in two different ways. First, risk groups that are not profitable are offered no insurance. Second, the insurer incentivizes individuals in profitable risk groups to reveal their private

[56] In Section 3 we explained that the model can also generate partial coverage by offering positive pooling contracts. However, our baseline economy doesn't generate a positive pooling contract for any risk group.

type by offering a menu featuring a less and a more comprehensive contract.[56] We now turn to consider the individual roles of Medicaid, administrative costs and private information in determining the pricing and comprehensiveness of coverage for risk groups that are offered insurance.

Table 9 reports the LTCI take-up rates, fractions of NH costs covered, and loads on good and bad risk types in the Baseline economy and the other three economies. The table reports the average value of each statistic and and a break down by private information type. Removing administrative costs produces savings to the insurer that get passed through to consumers in the form of higher comprehensiveness of coverage and lower loads. Allowing the insurer to directly observe private information type also increases comprehensiveness but average loads increase. Under full information, the insurer is able to extract the entire surplus, and both good and bad risks have binding participation constraints. Note that the load on bad risks increases substantially from -0.012 to 0.316 and this group's LTCI take-up rate falls. The intuition for this finding can be found in Arrow (1963) who demonstrates that the amount of insurance available to those with high risk exposures declines if insurance markets open after their risk exposure is observed.

Medicaid, acts like a competitor to private insurance and removing it also allows the private issuer to extract more rents from individuals. This is reflected in higher loads on average and for each private information type in the No Medicaid economy. The pricing distortion is particularly large for good types who face a load of 0.717 in the No Medicaid economy versus 0.593 in the Baseline economy. However, they are compensated somewhat by higher comprehensiveness of coverage which increases by 9 percentage points. Take-up rates are very high in the No Medicaid economy. Lacking the outside option of Medicaid, 90.6% of good risk and bad risk types purchase LTCI. Finally, note that bad risk types get a relatively good deal in this economy because the insurer is constrained in the amount of rents he can extract from them by the incentive compatibility constraint. The loads for bad risks are only 0.167 and their contracts cover 82.5% of NH expenses on average.

Brown and Finkelstein (2008) and Ameriks et al. (2016) use a different strategy to assess the roles of high loads, incomplete coverage and Medicaid in accounting for low LTCI take-up rates. Both of these papers specify contracts exogenously and consider counterfactuals in which individuals are offered full insurance against NH risk at an actuarially-fair price. Brown and Finkelstein (2008) find that only the top one-third of individuals, when ranked by wealth, purchase a full-coverage actuarially-fair LTCI policy when Medicaid is present. In other words, in their model Medicaid crowds-out the demand for LTCI by individuals in the bottom two-thirds of the wealth distribution. In our model contracts are endogenous and the insurer responds to Medicaid not only by adjusting the fraction of individuals who it insures but also by adjusting the comprehensiveness and pricing of the contracts.

As we now show, the crowding-out effect of Medicaid is much smaller when the insurer's optimal contracting problem is modeled. To illustrate this point, consider a version of our Baseline economy in which the two supply-side frictions — private information and administrative costs — are removed. Medicaid is present with the consumption floor set at the baseline level. Insurance is not actuarially fair in this scenario, the average load is 0.36, due to the fact that the insurer is a monopolist. Nevertheless, 61% of individuals purchase LTCI. Table 10 reports LTCI take-up rates, comprehensiveness of coverage and average loads by wealth and frailty quintiles in this alternative economy. LTCI take-up rates are 100% in

Table 9: LTCI take-up rates, comprehensiveness and individual loads by private type in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information economies

Scenario Description	Baseline	No Admin. Costs $\lambda = 1,\, k = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information $\theta^i_{f,\mathbf{w}}$ public
Average				
LTCI take-up rate	0.097	0.610	0.906	0.375
Fraction of NH costs covered	0.582	0.629	0.662	0.839
Load	0.415	0.333	0.557	0.483
Good risks (θ^g)				
LTCI take-up rate	0.097	0.609	0.906	0.524
Fraction of NH costs covered	0.506	0.547	0.596	0.839
Load	0.593	0.538	0.717	0.484
Bad risks (θ^b)				
LTCI take-up rate	0.099	0.613	0.906	0.012
Fraction of NH costs covered	0.753	0.816	0.825	0.848
Load	-0.012	-0.162	0.167	0.316

The fraction of NH costs covered is the average indemnity divided by the medical expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance.

wealth quintiles 3–5. Medicaid crowds out most private insurance in wealth quintile 2 and all private insurance in quintile 1. Wealth quintile 2 is particularly interesting because the load on insurance for this group is only 0.16 and thus reasonably close to the actuarially-fair benchmark. Yet, LTCI only covers half of the loss. These individuals are not interested in a full-coverage private LTCI product because for some values of the demand shock they will qualify for Medicaid NH benefits.[57] Indeed, 96% of individuals in wealth quintile 2 prefer to rely exclusively on Medicaid.

In contrast to individuals in the lower wealth quintiles, those in quintile 4 receive extensive coverage (88% of the loss) and those in quintile 5 receive nearly full coverage with insurance covering 97% of the loss. For the latter group, the chance of receiving Medicaid NH benefits is particularly low and full coverage is attractive to them. This final property of the model is related to Ameriks et al. (2016). They find that 66% of individuals in a sample of affluent individuals with median wealth of $543,000 have demand for an ideal state-contingent LTCI product that is priced in an actuarially-fair manner. However, only 22% of their respondents hold LTCI and they refer to this as a "LTCI puzzle." For purposes of comparison, in our baseline model, average wealth in wealth quintile 5 is $695,000 and average wealth in quintile 4 is $304,000.[58] In our baseline economy, individuals in wealth quintiles 4–5 have LTCI take-up rates of 14% and 21%, respectively. Thus, we find that the LTCI puzzle that Ameriks et al. (2016) document for wealthy individuals can be attributed to supply-side distortions

[57] Recall that the optimal contracts have this same partial coverage property in the simple model.

[58] Both figures are expressed in terms of year 2000 dollars.

Table 10: LTCI take-up rates, comprehensiveness, and individual loads in the economy with no private information and no administrative costs.

	Wealth Quintile				
	1	2	3	4	5
LTCI take-up rates	0.00	0.04	1.00	1.00	1.00
Fraction of loss covered	NA	0.50	0.61	0.88	0.97
Average load	NA	0.16	0.30	0.41	0.38
	Frailty Quintile				
	1	2	3	4	5
LTCI take-up rates	0.75	0.67	0.62	0.57	0.44
Fraction of loss covered	0.85	0.83	0.80	0.77	0.80
Average load	0.42	0.42	0.41	0.40	0.43

NA denotes cases where the denominator is zero.

induced by private information and administrative costs.

We have focused on these two examples because they are the most relevant to our analysis. However, it is common practice in the literature to abstract from the contract design problem of the issuer when modeling the LTCI market. Some recent examples include Lockwood (2016) who analyzes optimal saving and bequests in a setting with exogenously specified LTCI and Mommaerts (2015) and Ko (2016) who analyze the informal care market under the assumption that an alternative option is an exogenously specified LTCI contract. It is conceivable that modeling the supply-side of the LTCI market would provide new insights into saving decisions of the old in the presence of bequest motives and their demand for informal care.

6.4.1 Profits

In Section 2 we documented that profits in the U.S. private LTCI market are low. Profits are also low in our Baseline economy. They are 2.3% of revenues. The left panel of Figure 7, which reports the distribution of profits by frailty and PE quintiles in the Baseline economy, reveals that most profits come from insuring healthy rich individuals as most of the other risk groups are rejected and profits are thus zero. Medicaid, administrative costs and private information all work to reduce profits. Medicaid, however, has the largest impact. When it is removed profits rise to 28.5% of revenues.[59] The right panel of Figure 7 shows the distribution of profits by frailty and PE quintiles in the No Medicaid economy. In this economy, in contrast to the Baseline, the insurer generates most of his profits from the poor. Profits fall monotonically with permanent earnings and do not vary as much with frailty. Medicaid has a large effect on profits for two reasons. First, Medicaid's presence dramatically reduces the fraction of profitable risk groups and when Medicaid is removed the fraction of zero-profit rejected pools declines. Second, as we explained above in the discussion of loads,

[59]Profits are 15.2% of revenues in the No Administrative Costs economy and 9.8% of revenues in the Full Information economy.

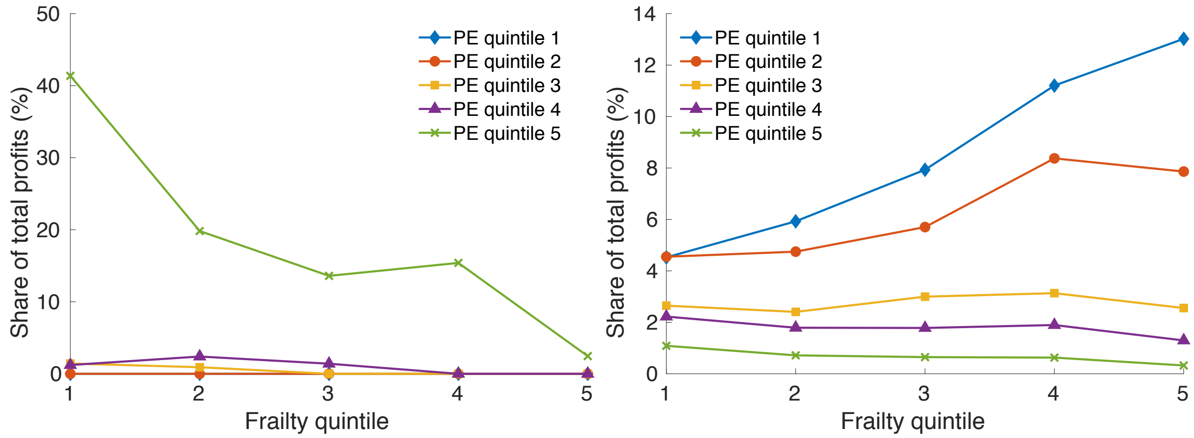


Figure 7: Profits Baseline and No Medicaid Specifications.
The left (right) panel reports the distribution of profits by frailty and PE quintiles in the Baseline (No Medicaid) economy. Note the difference in scale of the two plots.

Medicaid also substantially lowers profit margins from risk groups that are getting a positive amount of insurance.

6.5 Robustness

We start by considering the robustness of our finding that optimal menus that feature choice are rare. The parameter ψ plays a central role in determining the costs of cross-subsidization from good to bad risk types and ultimately the fraction of risk groups that are offered an optimal menu that provides them with the choice of either positive insurance or no insurance. In our setting with a monopoly insurer, premia from good types are used to cross-subsidize premia for bad types. If ψ is reduced a smaller fraction of good types is available to provide the subsidies and it becomes more likely that the optimal menus include a $(0, 0)$ contracts. To explore the quantitative significance of this effect, we reparametrized the model with ψ reduced from its baseline value of 0.709 to 0.609. When ψ is 0.609 the fraction of individuals that are offered the choice of $(0, 0)$ contract and opt for it increases from 0.11% to 6.3% and the number of no-trade contracts falls to 82%. Even though choice becomes relatively more important, variation along the extensive margin is still the main reason why LTCI take-up rates are low. One important difference between this scenario and the baseline parametrization is that it is much easier to detect adverse selection by comparing NH entry frequencies of LTCI holders and non-holders in this scenario. For instance, the fraction of LTCI holders who enter a NH (conditional on survival) is now larger, at 0.44, than the fraction of non-holders who enter a NH (0.40). Thus, the model with a lower value of ψ no longer accounts for the adverse selection correlation puzzle.

Another important difference is that the model with the lower value of ψ exhibits too little dispersion in private information as compared to our data. The coefficient of variation of private information produced by the model falls from 0.94 in the baseline to 0.86 when $\psi = 0.609$. Recall, that the target for our baseline value of ψ is the coefficient of variation of self-reported NH entrance probabilities. Our data measure omits responses of 0, 1/2 and 1. If some/all of these responses are included, the coefficient of variation for self-reported

NH risk is even larger than 0.94. In this sense, our strategy for setting ψ in the baseline parametrization is conservative. Apart from these two differences, the performance of the model with a lower ψ is similar to our baseline parametrization. In particular, this version of the model is able to match the pattern of LTCI take-up rates and NH entry rates by frailty and wealth.

Our strategy for parametrizing the model used particular data facts to pin down the scale of Medicaid NH benefits and administrative costs. We explored how the results change if we assign a more prominent role to each of these factors. One experiment we performed was to increase the scale of the Medicaid consumption floor by a factor of 1.76. This value lies at the upper end of values used in previous studies[60] Private information and administrative costs continue to have a large impact on LTCI take-up rates of affluent individuals even with the higher consumption floor. For instance, LTCI take-up rates increase by 50% or more if the private information distortion is removed.

Recall that LTCI take-up rates are very high in the Full Information economy. We have explored whether is is possible to reparametrize this version of the model to reproduce the LTCI take-up rates in the data. We need significantly higher administrative costs (49% of premia instead of 33%) to get the average take-up rate to match the data. However, this parametrization cannot generate the empirical pattern of LTCI take-up rates at alternative frailty quintiles among more affluent individuals no matter how we adjust NH entry probability distribution.

Finally, we have considered how well the model performs if it is reparametrized under the assumption that administrative costs are absent. This version of the model fails to produce low LTCI take-up rates among affluent risk-groups with high frailty levels. For instance, the model predicts that LTCI take-up rates of individuals in wealth and frailty quintile 5 are nearly 1 while they are only 0.1 in our data. More complete details about these robustness checks can be found in Section 5.2 of the appendix.

Our model has abstracted from several features of the U.S. LTCI market. Most notably in recent years regulators in this industry have required that issuers add markups, called "pricing margins", to the price of their initial premia to reduce the probability of future premium increases due to intertemporal risk. We don't model pricing margins. In the model neither individuals nor issuers face aggregate uncertainty about interest rates, mortality rates or LTCI take-up rates and there is no reason to provision for it. Pricing margins could compound the problem of adverse selection.

We have also abstracted from moral hazard. In the 1980s and early 1990s LTCI insurers were not concerned about moral hazard because they felt that individuals given the choice would prefer not to be institutionalized. However, as coverage has been expanded to provide home care benefits, insurers have had to allocate more resources to claims management to ascertain that individuals have an ongoing need for LTC services. In this sense, high administrative costs in this market may partially reflect moral hazard.

[60] See Kopecky and Koreshkova (2014) for a summary of previous studies.

7 Concluding Remarks

We have found that the standard adverse selection model with a single source of private information is a good empirical model of the U.S. LTCI market when augmented to allow for screening along the extensive margin. Our finding that lack of gains from trade between insurers and entire risk groups is quantitatively important may apply to other insurance markets. For instance, the U.S. individual health care insurance market prior to the Affordable Care Act (ACA), shared many features with the U.S. LTCI market: take-up rates were low, rejections were common, and loads were high for insured individuals. Underwriting is also used as a screening device in life and disability insurance markets.

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Online Appendix to ‘Old, frail, and uninsured: Accounting for features of the U.S. long-term care insurance market.’

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1 Additional theoretical results and discussion

1.1 Additional material for Section 3.1 of the paper

The insurer's problem in the one-period model discussed in Section 3.1 of the paper (administrative costs but no Medicaid) is

$$\max_{\pi^g,\iota^g,\pi^b,\iota^b} \psi\Big\{\pi^g - \theta^g\big[\lambda\iota^g + kI(\iota^g > 0)\big]\Big\} + (1-\psi)\Big\{\pi^b - \theta^b\big[\lambda\iota^b + kI(\iota^b > 0)\big]\Big\}, \tag{1}$$

subject to

$$(PC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,0,0) \geq 0, \quad i \in \{g,b\}, \tag{2}$$

$$(IC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,\pi^j,\iota^j) \geq 0, \quad i,j \in \{g,b\},\ i \neq j, \tag{3}$$

where $\lambda \geq 1$ captures variable costs and $k \geq 0$ captures fixed costs.

Denote consumption of an individual with risk type i as c^i_{NH} in the NH state and c^i_o otherwise. An individual's utility function is

$$\begin{aligned} U(\theta^i,\pi^i,\iota^i) =& \theta^i u(\omega - \pi^i - m + \iota^i) + (1-\theta^i)u(\omega - \pi^i), \\ =& \theta^i u(c^i_{NH}) + (1-\theta^i)u(c^i_o), \end{aligned} \tag{4}$$

and the associated marginal rate of substitution between premium and indemnity is

$$\frac{\partial\pi}{\partial\iota}(\theta^i) = -\frac{U_\iota(\cdot)}{U_\pi(\cdot)} = \frac{\theta^i u'(c^i_{NH})}{\theta^i u'(c^i_{NH}) + (1-\theta^i)u'(c^i_o)} \equiv MRS(\theta^i,\pi^i,\iota^i). \tag{5}$$

Assume that the utility function has the property that $MRS(\theta^i, \pi^i, \iota^i)$ is strictly increasing in $\theta^i, i \in \{g, b\}$. This is true, for example, if $u'(\cdot) > 0$. Under this assumption, which is referred to as the single crossing property, any menu of contracts that satisfies incentive compatibility will have the property that if $\theta^{i'} > \theta^i$ then $\pi^{i'} \geq \pi^i$ and $\iota^{i'} \geq \iota^i$.

At the optimal menu, Equation (2) will bind for the good types and Equation (3) will bind for the bad types. If the optimal menu features positive insurance than it will also satisfy the two first-order conditions

$$\psi MRS(\theta^g, \pi^g, \iota^g) + (1-\psi)\left[\frac{U_\pi(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)} MRS(\theta^g, \pi^g, \iota^g) + \frac{U_\iota(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)}\right] = \lambda\psi\theta^g, \quad (6)$$

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b. \quad (7)$$

When $\lambda = 1$ and $k = 0$, the equilibrium menu is standard. This means that it will always be a separating one with bad types receiving full insurance. When $\lambda > 1$ and/or $k > 0$, the optimal menu may be a pooling menu. In the case of $k > 0$ and $\lambda = 1$ the only type of pooling menu that can arise is a no-trade menu. When $\lambda > 1$ both no-trade and pooling menus featuring positive insurance can arise. An optimal pooling menu featuring positive insurance must satisfy

$$MRS(\theta^g, \pi, \iota) = \lambda\eta, \quad (8)$$

$$U(\theta^g, \pi, \iota) - U(\theta^g, 0, 0) = 0. \quad (9)$$

These two equations can be derived using from Equations (2), (3), (6) and (7), where $\pi = \pi^g = \pi^b$ and $\iota = \iota^b = \iota^b$.

For simplicity, the good types' contract in Figures 1a and 1b in the paper are illustrated as the optimal pooling contract. Rearranging the first-order conditions, one can show that Equation (6) is equivalent to

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda\left[\frac{\psi\theta^g + (1-\psi)\theta^b A}{\psi + (1-\psi)B}\right], \quad (10)$$

where $A \equiv U_\iota(\theta^b, \pi^g, \iota^g)/U_\iota(\theta^b, \pi^b, \iota^b)$ and $B \equiv U_\pi(\theta^b, \pi^g, \iota^g)/U_\pi(\theta^b, \pi^b, \iota^b)$. The figure corresponds to cases where A and B are approximately 1.

Proposition 1. *If $\lambda > 1$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. First, note that the slope of the indifference curve at the full insurance level of indemnity always equals θ^i or

$$MRS(\theta^i, \pi^i, m) = \left.\frac{\theta^i u'(\omega - \pi^i - m + \iota^i)}{\theta^i u'(\omega - \pi^i - m + \iota^i) + (1-\theta^i)u'(\omega - \pi^i)}\right|_{\iota^i = m} = \theta^i, \quad (11)$$

for all π^i. Second, note that the slope of the indifference curve declines with the level of indemnity or

$$\frac{\partial MRS(\theta^i, \pi^i, \iota^i)}{\partial \iota^i} = \frac{\theta^i(1-\theta^i)u''(c_{NH})u'(c_o)}{[\theta^i u'(c_{NH}) + (1-\theta^i)u'(c_o)]^2} < 0. \quad (12)$$

The good type is always under-insured, regardless of whether the optimal contract is pooling or separating. To see this for the optimal pooling contract (π^p, ι^p), combine Equation (8) with Equation (11) to obtain the following inequality

$$MRS(\theta^g, \pi^p, \iota^p) = \lambda\eta > \theta^g = MRS(\theta^g, \pi^p, m),$$

which holds when $\lambda > 1$ since $\eta = \psi\theta^g + (1-\psi)\theta^b \geq \theta^g$. Then it follows from Equation (12) that $\iota^p < m$. If instead the equilibrium is separating then combine the expression for Equation (10) and Equation (11) to obtain

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda \left[\frac{\psi\theta^g + (1-\psi)\theta^b A}{\psi + (1-\psi)B}\right] > \theta^g = MRS(\theta^g, \pi^g, m),$$

where $A \equiv U_\iota(\theta^b, \pi^g, \iota^g)/U_\iota(\theta^b, \pi^b, \iota^b)$ and $B \equiv U_\pi(\theta^b, \pi^g, \iota^g)/U_\pi(\theta^b, \pi^b, \iota^b)$. The inequality holds for $\lambda \geq 1$ since under the single-crossing property any incentive compatible separating contract must be such that $\pi^g < \pi^b$ which implies that

$$A = \frac{\theta^b u'(c^g_{NH})}{\theta^b u'(c^b_{NH})} > \frac{\theta^b u'(c^g_{NH}) + (1-\theta^b)u'(c^g_o)}{\theta^b u'(c^b_{NH}) + (1-\theta^b)u'(c^b_o)} = B,$$

where $c^i_{NH} = \omega - m + \iota^i - \pi^i$ and $c^i_o = \omega - \pi^i$. Thus, from Equation (12), $\iota^g < m$. Finally, combine Equation (7) and Equation (11) to obtain

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b > \theta^b = MRS(\theta^b, \pi^b, m),$$

which holds when $\lambda > 1$. So, from Equation (12), $\iota^b < m$. □

Proposition 2. *There will be no trade, i.e., the optimal menu will consist of a single* $(0, 0)$ *contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda\theta^b, \tag{13}$$
$$MRS(\theta^g, 0, 0) \leq \lambda\eta, \tag{14}$$

both hold.

Proof. Assume that u is strictly concave so that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and $k = 0$. First, we will show that if Equations (13) and (14) hold then the optimal menu will be a single $(0, 0)$ contract.

Part 1: We will show that if Equation (13) holds then the optimal menu must be a pooling menu. Suppose the optimal menu features a contract for the good types (π^g, ι^g) with $\iota^g \geq \pi^g \geq 0$ and a contract for the bad types (π^b, ι^b) with $\iota^b \geq \pi^b \geq 0$. The following inequalities hold:

$$\theta^b\lambda \geq MRS(\theta^b, 0, 0) \geq MRS(\theta^g, \iota^g, \pi^g). \tag{15}$$

The first inequality is Equation (13) and the second follows from $u'(\cdot) > 0$, $\iota^g \geq \pi^g \geq 0$, and $\theta^b > \theta^g$. By single-crossing we have that $\pi^b \geq \pi^g$ and $\iota^b \geq \iota^g$. This together with the fact that u is strictly concave means that

$$\pi^b - \pi^g \leq MRS(\theta^g, \iota^g, \pi^g)(\iota^b - \iota^g). \tag{16}$$

Combining (15) and (16) yields

$$\pi^b - \pi^g \leq \theta^b \lambda(\iota^b - \iota^g), \tag{17}$$

and rearranging gives

$$\pi^g - \theta^b \lambda \iota^g \geq \pi^b - \theta^b \lambda \iota^b). \tag{18}$$

However, this implies that giving the bad types (π^g, ι^g) which they value as equal to (π^b, ι^b) does not reduce (and may increase) profits. The only way this can be is if $(\pi^g, \iota^g) = (\pi^b, \iota^b) \equiv (\pi, \iota)$.

Part 2: We will show that if Equation (14) holds then the optimal pooling contract must be a no-trade, $(0, 0)$ contract. If $\pi > 0$ and $\iota > 0$ then the following inequalities hold:

$$\eta\lambda \geq MRS(\theta^g, 0, 0) > MRS(\theta^g, \iota, \pi). \tag{19}$$

The first inequality is Equation (14) and the second follows from $u'(\cdot) > 0$ and $\iota \geq \pi > 0$. Thus (π, ι) does not satisfy the first-order optimality conditions for a pooling contract. The pooling contract must be $(0, 0)$.

Now, we will show that if the optimal menu is a single $(0, 0)$ contract then Equations (13) and (14) hold. Suppose Equation (13) does not hold. Then there exists a menu that gives $(0, 0)$ to good types and a small amount of insurance to bad types with a contract (π^b, ι^b) that satisfies

$$MRS(\theta^b, \pi^b, \iota^b) \geq \lambda\theta^b,$$

and with a premium chosen such that $U(\theta^b, \pi^b, \iota^b) = U(\theta^b, 0, 0)$. This menu satisfies all the constraints of the insurer's problem and delivers higher profits to the insurer. Thus the optimal menu can not consistent of a single $(0, 0)$ contract.

Suppose Equation (14) does not hold. Then there exists a pooling contract (π, ι) that gives a small amount of insurance to both types and satisfies

$$MRS(\theta^g, \pi, \iota) \geq \lambda\eta,$$

with the premium such that $U(\theta^g, \pi, \iota) = U(\theta^g, 0, 0)$. This menu satisfies all the constraints of the insurer's problem and delivers higher profits to the insurer. Thus the optimal menu can not consistent of a single $(0, 0)$ contract.

□

Intuition: No trade equilibria occur when the amount individuals are willing to pay for even a small positive separating or pooling equilibrium is less than the amount required to provide nonnegative profits to the insurer. Condition (13) rules out profitable separating menus where only bad types have positive insurance, such as the one illustrated in Figure 1e in the paper. Condition (14) rules out profitable pooling and separating menus where both types are offered positive insurance.

1.2 Additional material for Section 3.2 of the paper

The insurer's problem in the version of the one-period model discussed in Section 3.2 of the paper (Medicaid but no administrative costs) is

$$\max_{\pi^g,\iota^g,\pi^b,\iota^b} \psi\Big\{\pi^g - \theta^g\iota^g\Big\} + (1-\psi)\Big\{\pi^b - \theta^b\iota^b\Big\}, \tag{20}$$

subject to

$$(PC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,0,0) \geq 0, \quad i \in \{g,b\}, \tag{21}$$
$$(IC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,\pi^j,\iota^j) \geq 0, \quad i,j \in \{g,b\},\ i \neq j, \tag{22}$$

where an individual's utility function is

$$U(\theta^i,\pi^i,\iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c^i_{NH}(\omega)) + (1-\theta^i)u(c^i_o(\omega))\Big] dH(\omega), \tag{23}$$

with

$$c^i_o(\omega) = \omega - \pi^i, \tag{24}$$
$$c^i_{NH}(\omega) = \omega + TR(\omega,\pi^i,\iota^i) - \pi^i - m + \iota^i. \tag{25}$$

The Medicaid transfer is defined by Equation (4) in the paper.

We first show that, if $u'(\cdot) > 0$, the single-crossing property continues to obtain when Medicaid is present and the endowment is stochastic.

Lemma 1. ***(Single-crossing Property)*** *If $u'(\cdot) > 0$, the single-crossing property holds when the endowment is stochastic and Medicaid is present with $\underline{c}_{NH} > 0$.*

Proof. Denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi,\iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (4) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$. The proof shows that $\frac{\partial MRS(\theta,\pi,\iota)}{\partial\theta} > 0$ for all $\pi,\iota \in \mathbb{R}^+$. Recall that

$$MRS(\theta,\pi,\iota) = -\frac{U_\iota(\theta,\pi,\iota)}{U_\pi(\theta,\pi,\iota)},$$

where

$$U_\iota(\theta,\pi,\iota) = \theta \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega) > 0, \tag{26}$$
$$U_\pi(\theta,\pi,\iota) = -U_\iota(\theta,\pi,\iota) - (1-\theta)B < 0, \tag{27}$$

with $B \equiv \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega) > 0$. Differentiating the MRS with respect to θ yields

$$\frac{\partial MRS}{\partial\theta} = -\frac{U_{\iota\theta}U_\pi - \theta U_\iota U_{\pi\theta}}{U_\pi^2}, \tag{28}$$

where

$$U_{\iota\theta} = \theta^{-1} U_\iota > 0, \tag{29}$$

$$U_{\pi\theta} = -U_{\iota\theta} + B = -\theta^{-1} U_\iota + B, \tag{30}$$

and the arguments are omitted to save space. Using Equations (26)–(30) it is easy to show that

$$\frac{\partial MRS}{\partial \theta} = \frac{B U_{\iota\theta}}{\theta U_\pi^2} > 0.$$

□

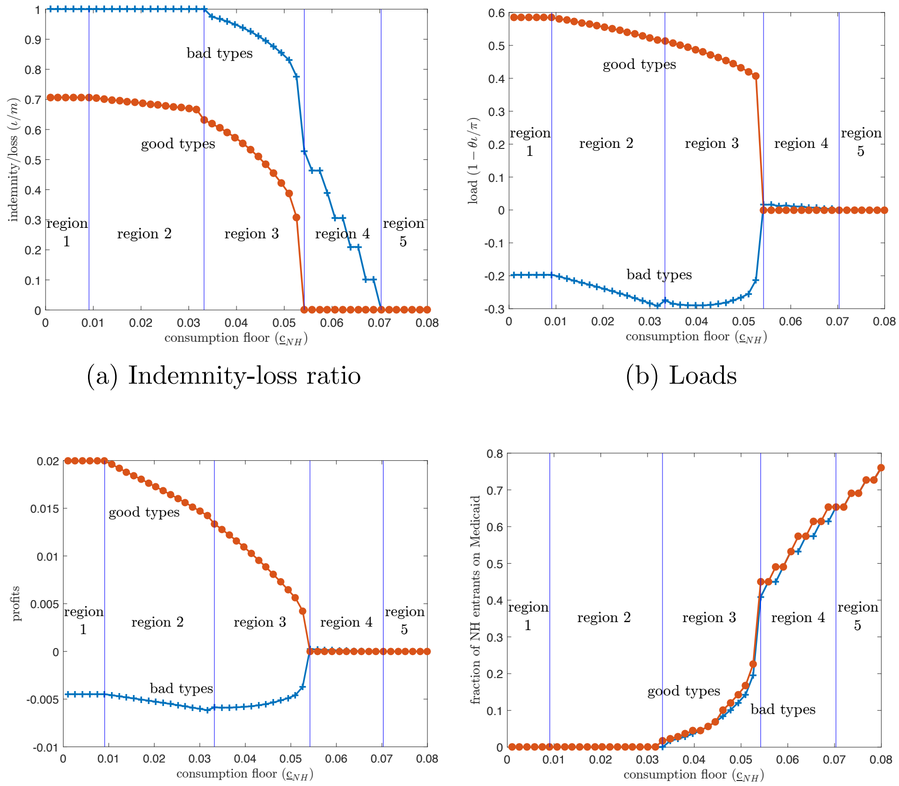
Figure 1 illustrates how the optimal contracts, profits and Medicaid take-up rates evolve as the Medicaid consumption floor, $\underline{c}_{NH}$, is increased from zero in the setup with endowment uncertainty. The figure is divided into 5 distinct regions. In region 1, the consumption floor is so low that even if an individual has no private LTCI and the smallest realization of the endowment he will not qualify for Medicaid. In this region, Medicaid has no effect on the optimal contracts. In region 2, Medicaid influences the contracts even though, in equilibrium, neither type receives Medicaid transfers. In this region, Medicaid has a similar effect to that illustrated in Figure 2b in the paper. For some realizations of the endowment, good types qualify for Medicaid if the contract is $(0, 0)$. This tightens their participation constraint and the contract offered to them has to be improved. A better contract for good types tightens, in turn, the incentive compatibility constraint for bad types. The insurer responds by reducing premiums for both types, and the indemnity of the good types and loads on both types fall. Since Medicaid's presence has resulted in more favorable contracts for individuals, the insurer's profits fall. In region 3, Medicaid has the same effects as in region 2 but now, in addition, both types receive Medicaid benefits in equilibrium for some realizations of ω. As discussed above, the partial insurance of NH shocks via Medicaid results in optimal contracts that feature partial coverage and, in this region, both types have less than full private insurance. Proposition 3 provides a sufficient condition for this to occur.

Proposition 3. *If $\underline{\omega} < \underline{c}_{NH}$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. Denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi, \iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (4) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$.

We start by showing that if $\underline{\omega} < \underline{c}_{NH}$ then the optimal contract for any type $i \in \{b, g\}$, (π^i, ι^i), is such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$. Suppose instead that $\hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega}$. In this case, no one of type i is on Medicaid in equilibrium. The utility function, Equation (6) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1 - \theta^i) u(c_o)\Big] dH(\omega),$$



(a) Indemnity-loss ratio

(b) Loads

(c) Profits

(d) Fraction of NH entrants on Medicaid

Figure 1: Impact of varying the Medicaid consumption floor, $\underline{c}_{NH}$, on the indemnity-loss ratio, loads, profits, and the fraction of NH entrants on Medicaid when the endowment is stochastic.

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$MRS(\theta^i, \pi^i, \iota^i) = \frac{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega) + (1-\theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}.$$

Following the same proof strategy as that of Proposition 1 it is easy to show that if $\lambda \geq 1$ then $\iota^i \leq m$ for $i \in \{g, b\}$. However, since $\underline{\omega} < \underline{c}_{NH}$ we have

$$\underline{c}_{NH} + m - \iota^i + \pi^i \equiv \hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega} < \underline{c}_{NH},$$

which implies that $\iota^i - \pi^i > m$ and since $\pi^i > 0$ it must be that $\iota^i > m$, a contradiction. We have established that the equilibrium contract for each type $i \in \{g, b\}$ must be such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$.

If $\hat{\omega}(\pi^i, \iota^i) \geq \overline{\omega}$ then everyone of type i is on Medicaid in equilibrium and the utility function, Equation (6) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i) u(c_o)\Big] dH(\omega),$$

where $c_o = \omega - \pi^i$. In this case, $MRS(\theta^i, \pi^i, \iota^i) = 0$ for all (π^i, ι^i) and the optimal contract is $(0, 0)$.

We now establish that for $i \in \{b, g\}$, $\iota^i < m$ holds when $\hat{\omega}(\pi^i, \iota^i) \in (\underline{\omega}, \overline{\omega})$ by showing that $\iota^i \geq m$ leads to a contradiction. The utility function, Equation (6) in the paper, can be stated as

$$\begin{aligned} U(\theta^i, \pi^i, \iota^i) = &\int_{\underline{\omega}}^{\hat{\omega}(\pi^i,\iota^i)} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega) \\ &+ \int_{\hat{\omega}(\pi^i,\iota^i)}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega), \end{aligned}$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$\begin{aligned} MRS(\theta^i, \pi^i, \iota^i) &= \frac{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega) + (1-\theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}, \\ &= \left[1 + \frac{(1-\theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)}\right]^{-1}. \end{aligned}$$

If $\iota^i \geq m$ then $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$. To see this suppose that $MRS(\theta^i, \pi^i, \iota^i) \geq \theta^i$ which implies that

$$\begin{aligned} 1 + \frac{(1-\theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)} \leq \frac{1}{\theta^i} \Leftrightarrow& \\ \int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\overline{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega).& \end{aligned} \tag{31}$$

Since $\iota^i \geq m$, we have $c_{NH} = \omega - \pi^i + \iota^i - m \geq c_o = \omega - \pi^i$, which implies $u'(c_{NH}) - u'(c_o) \leq 0$. Equation (31) becomes

$$\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega) \leq 0, \tag{32}$$

which is a contradiction since $u'(c_0) > 0$ and $\underline{\omega} < \hat{\omega} < \overline{\omega}$.

Having established that $\iota^i \geq m$ implies $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$ for $i \in \{b, g\}$ the final step is to show that this condition violates the necessary conditions for an optimal contact. First consider an optimal pooling contract (π^p, ι^p). Note that $\theta^g < \lambda\eta$ since $\lambda \geq 1$ and $\eta = \psi\theta^g + (1-\psi)\theta^b > \theta^g$. So $MRS(\theta^g, \pi^p, \iota^p) < \lambda\eta$ when $\iota^p \geq m$. This is a contradiction because the optimal pooling contract must satisfy Equation (8). It follows that $\iota^p < m$.

Now consider an optimal separating contract. First, consider good types. Under the optimal contract it must be that $\theta^g < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$, since $\lambda \geq 1$ and due to single-crossing (established in Lemma 1) and incentive compatibility $\pi^g < \pi^b$ so

$$A = \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c^g_{NH}) dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c^b_{NH}) dH(\omega)} > \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c^g_{NH}) dH(\omega) + (1-\theta^b) \int_{\underline{\omega}}^{\bar{\omega}} u'(c^g_o) dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c^b_{NH}) dH(\omega) + (1-\theta^b) \int_{\underline{\omega}}^{\bar{\omega}} u'(c^b_o) dH(\omega)} = B,$$

where $c^i_{NH} = \omega - m + \iota^i - \pi^i$ and $c^i_o = \omega - \pi^i$. Hence $MRS(\theta^g, \pi^g, \iota^g) < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$ when $\iota^g \geq m$. This is a contradiction because the equilibrium contract for good types must satisfy Equation (6). It follows that $\iota^g < m$.

Second, consider bad types. We have established that if $\iota^b \geq m$ then $MRS(\theta^b, \pi^b, \iota^b) < \theta^b \leq \lambda\theta^b$ since $\lambda \geq 1$. This is a contradiction because the equilibrium contract for bad types must satisfy Equation (7). It follows that $\iota^b < m$. □

Recall that Proposition 1 showed that when the price of private insurance is high due to variable administrative costs incurred by the insurer, the optimal contracts will feature less than full insurance for both risk types. Similarly, Proposition 3 shows that when the implicit price of private insurance is high because individuals are at least partially covered by Medicaid than the optimal contracts will also feature less than full insurance.

In region 4 in the graphs in Figure 1, the consumption floor is so high that the good types, who's willingness to pay for private LTCI is lower than the bad types, choose to drop out of the private LTCI market. Notice that, even though the average loads are declining as the consumption floor increases, the load on bad types jumps up upon entry into this region. In regions 1–3, the contracts exhibit cross-subsidization with bad types benefiting from negative loads and good types facing positive loads. In region 4, the insurer is able to make a small amount of positive profits by offering a positive contract that is only attractive to the bad types. Finally, in region 5, Medicaid has a similar effect to that depicted in Figure 2c in the paper. The consumption floor is so large that there are no terms of trade that generate positive profits from either type. The insurer rejects applicants when the consumption floor is in this region as the optimal menus consist of a single $(0,0)$ contract.

Due to the non-convexities Medicaid creates, conditions (13) and (14) in Proposition 2 are no longer sufficient conditions for rejections to occur, and, although still necessary, are not very useful. Proposition 4 provides a stronger set of necessary conditions for rejections in the presence of Medicaid and a stochastic endowment.

Proposition 4. *If the optimal menu is a $(0,0)$ pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall \iota \in \mathbb{R}_+, \tag{33}$$

and

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall \iota \in \mathbb{R}_+. \tag{34}$$

Proof. The proposition is proved by showing that if the conditions don't hold, one can find a menu with at least one nonzero contract that satisfies all the constraints and delivers non-negative profits.

First, assume that condition (33) does not hold but that condition (34) does. If (33) does not obtain, there exists $\iota \in \mathbb{R}_+$ such that

$$U(\theta^b, \lambda\theta^b\iota, \iota) \geq U(\theta^b, 0, 0). \tag{35}$$

Give bad types $(\lambda\theta^b\iota, \iota)$ and good types $(0,0)$. Under this menu the insurer's profits are

$$\Pi = (1-\psi)\lambda\theta^b\iota + \psi 0 - (1-\psi)\lambda\theta^b\iota - \psi 0 = 0;$$

the participation constraint for the bad types, which is also their incentive compatibility constraint, holds by condition (35); the participation constraint of the good types is trivially satisfied; and the incentive compatibility constraint for the good types is satisfied since

$$U(\theta^g, \lambda\theta^b\iota, \iota) < U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0),$$

where the first inequality follows from the fact that $\eta < \theta^b$ and the second from condition (34).

Second, assume that condition (34) does not hold which means there exists $\iota \in \mathbb{R}^+$ such that

$$U(\theta^g, \lambda\eta\iota, \iota) \geq U(\theta^g, 0, 0). \tag{36}$$

Give both types $(\lambda\eta\iota, \iota)$. Under this pooling contract the insurer's profits are

$$\Pi = \lambda\eta\iota - \lambda\eta\iota = 0,$$

the participation constraint of the good types holds by condition (36), and the participation constraint for the bad types holds since condition (36) holds and U satisfies the single-crossing property established in Lemma (1). Note that the incentive compatibility constraints are trivially satisfied since both types get the same contract. □

If condition (33) fails, then one can find a profitable contract that bad types would take, and, if condition (34) fails, then one can find a profitable pooling contract that good types would take. The conditions are not sufficient because, while they rule out profitable pooling contracts and separating contracts where good types get no insurance, they do not rule out separating contracts where both types get positive insurance. Absent Medicaid, there can never exist a separating contract that increases profits if the optimal pooling contract is $(0, 0)$. However, the non-convexities introduced by Medicaid break this property. As a result, even when the optimal pooling contract generates negative profits, a profitable separating contract might still exist.

Figure 1 highlights some important distinctions between our model, where contracts are optimal choices of an issuer, and previous research by, for instance, Brown and Finkelstein (2008), Mommaerts (2015), and Ko (2016), who model demand-side distortions in the LTCI market but set contracts exogenously. In regions 2 and 3, notice that Medicaid's presence only impacts the pricing and coverage of the optimal private contracts. In these regions, the insurer responds to the reduced demand for private LTCI by adjusting the terms of the contracts but still offers positive insurance. In contrast, in regions 4 and 5, Medicaid's presence also impacts the fraction of individuals who have any private LTCI. Notice that the Medicaid recipiency rates of both types increase as the consumption floor is increased in these regions. This means that, even though good types do not have LTCI in region 4 and no individuals have it in region 5, Medicaid is covering their NH costs only for a subset of the endowment space. For some realizations of ω, they self-insure. Thus, in these regions, Medicaid is crowding-out demand for private LTCI despite providing only incomplete coverage itself. This crowding-out effect is also present in models with exogenous contracts, however, the effects of Medicaid on the terms of positive contracts is not. Thus, allowing the insurer to adjust the contracts in response to the presence of Medicaid is important because, if the terms of the contracts cannot adjust, then the crowding-out effect of Medicaid on the size of the LTCI market will be overstated.

1.3 Varying Rejection Rates across Risk Groups

The analysis in Sections 3.1 and 3.2 of the paper focuses on the problem of an insurer that offers insurance to a single risk group. We now turn to describe how the extent of rejections changes as we vary observable characteristics of individuals. This discussion provides intuition for the results found using the quantitative model which features an environment with a rich structure of public information and thus multiple risk groups.

One data fact we want the model to account for is that those with lower wealth have lower LTCI take-up rates. An explanation for this observation is that risk groups with low expected endowments are more likely to be rejected by the insurer due to Medicaid. The following proposition formalizes this claim.

Proposition 5. *When $\overline{\omega} - m \leq \underline{c}_{NH}$, the possibility of rejection in equilibrium increases if the distribution of endowments on $[\underline{\omega}, \overline{\omega}]$ is given by $H_1(\cdot)$ instead of $H(\cdot)$ where $H_1(\cdot)$ is first-order stochastically dominated by $H(\cdot)$.*

Proof. It is useful to express Equations (33)–(34) as

$$U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0) < 0, \quad \forall \iota \in \mathbb{R}^+, i \in \{g, b\}, \tag{37}$$

where

$$U(\theta^i, \pi^i, \iota) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] dH(\omega),$$

with

$$\pi^i = \begin{cases} \lambda\theta^b\iota, & \text{if } i = b, \\ \lambda\eta\iota, & \text{if } i = g. \end{cases}$$

Without loss of generality, assume that $m \geq \iota \geq \pi > 0$.

Let $\Delta U(H)$ and $\Delta U(H_1)$ represent $U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0)$ when the endowment distribution is given by $H(\cdot)$ and $H_1(\cdot)$, respectively. Then

$$\Delta U(H) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH(\omega),$$

and

$$\Delta U(H_1) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH_1(\omega),$$

where

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] - \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - m)) + (1-\theta^i) u(\omega)\Big].$$

If $\tilde{u}(\omega)$ is non-decreasing then $\Delta U(H) \geq \Delta U(H_1)$ and rejections are weakly more likely under H_1 than H. When $\overline{\omega} - m \leq \underline{c}_{NH}$ we have

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] - \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i) u(\omega)\Big],$$

and

$$\frac{d\tilde{u}(\omega)}{d\omega} = \begin{cases} \theta^i u'(\omega - \pi^i - m + \iota) + (1-\theta^i)[u'(\omega - \pi) - u'(\omega)], & \omega - \pi^i + \iota > \underline{c}_{NH}, \\ (1-\theta^i)[u'(\omega - \pi^i) - u'(\omega)], & \omega - \pi^i + \iota \leq \underline{c}_{NH}. \end{cases}$$

It is easy to see that $\frac{d\tilde{u}(\omega)}{d\omega} > 0$.

□

It immediately follows from Proposition 5 that the possibility of rejections increases if the expected endowment decreases when $\overline{\omega} - m \leq \underline{c}_{NH}$. When $\overline{\omega} - m > \underline{c}_{NH}$, decreasing the expected endowment may also lead to an increased possibility of rejection. However, in this case, it is also possible that the likelihood of rejections goes down since, absent Medicaid, lowering an individual's endowment raises his demand for insurance.

We also want the model to account for the fact that LTCI take-up rates are declining in frailty and that insurers are more likely to reject frail individuals. In the quantitative model, individuals vary by endowments and frailty, both of which are observable by the insurer, and the distribution of private information varies across these observable types. The following proposition shows two ways of varying the distribution of private information with frailty to generate an increasing possibility of rejection. Note that the proof is for the no Medicaid case, although, as the quantitative results illustrate, the proposition holds even when Medicaid is present.

Proposition 6. *When $\lambda > 1$ and θ^b is sufficiently close to 1, the possibility of rejection in equilibrium increases if:*

1. *θ^b increases;*
2. *θ^b increases and θ^g decreases such that the mean NH entry probability $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$ does not change.*

Proof. Without Medicaid, rejection will occur in equilibrium iff

$$f^b(\theta^b) \equiv \lambda\theta^b - MRS(\theta^b, 0, 0) \geq 0, \tag{38}$$

and

$$f^g(\theta^g, \theta^b) \equiv \lambda\eta - MRS(\theta^g, 0, 0) \geq 0, \tag{39}$$

where $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$.

1. Differentiating f^b with respect to θ^b yields

$$\frac{f^b(\theta^b)}{d\theta^b} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^b \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1-\theta^b)\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2}. \tag{40}$$

When $\theta^b = 1$, Equation (40) is positive since

$$\frac{f^b(\theta^b)}{d\theta^b}|_{\theta^b=1} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)}{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega)},$$

$\lambda \geq 1$ and $u'(\omega) < u'(\omega - m)$ for $m > 0$. It is easy to see that Equation (40) is increasing in θ^b. Thus if θ^b is sufficiently close to 1, increasing θ^b will increase f^b. Differentiating f^g with respect to θ^b yields

$$\frac{f^g(\theta^g, \theta^b)}{d\theta^b} = \lambda(1 - \psi) > 0.$$

Thus increasing θ^b increases f^g.

2. The proof that f^b is increases is the same as in 1 since f^b does not depend on θ^g. The first term of f^g does not change. The second term only depends θ^g and differentiating it with respect to θ^g yields

$$\frac{dMRS(\theta^g, 0, 0)}{d\theta^g} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m) dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega) dH(\omega)}{[\theta^g \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m) dH(\omega) + (1 - \theta^g) \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega) dH(\omega)]^2} > 0.$$

So f^g increases when θ^g declines.

□

Proposition 6 presents two ways to increase the possibility of a no-trade equilibrium occurring. Either of the two ways can, in theory, be used to generate decreasing LTCI take-up rates with frailty. If, as in way 1, only θ^b increases then both the mean and the dispersion of the NH entry probabilities will increase. However, way 2 states that increasing the dispersion of entry probabilities while holding the mean fixed by varying both θ^b and θ^g can also generate increased rejection rates. In short, to generate an increase in rejection rates with frailty, both ways require an increase in the dispersion in NH entry probabilities with frailty. However, way 1 also requires an increase in the mean.

The fact that both ways of increasing the rejection rates requires an increase in dispersion of private information is consistent with the empirical findings of Hendren (2013) that adverse selection is more severe among individuals that are more likely to rejected by LTC insurers. In addition, we show in Section 4 in the paper that the increase in dispersion is consistent with the pattern of NH entry probabilities in the data, but the increase in the mean implied by case 1 is inconsistent. Thus both θ^b and θ^g must vary with frailty, as in case 2, to generate patterns of both LTCI take-up rates and NH entry probabilities that are consistent with the data. Note that Hendren (2013) also presents a theory of how private information can generate no-trade equilibria (rejections). His mechanism, however, is different from ours. We generate no-trade equilibria by modeling administrative costs on the insurer and Medicaid. In his model, there is a continuum of private types and he allows there to be a positive mass of individuals who have probability 1 of incurring the loss. He shows that, under this assumption, the presence of private information can lead to no-trade equilibria. To activate his mechanism in our model with 2 private types we would have to assume that $\theta^b = 1$. He also shows that the possibility of rejections increases as the magnitude of private information increases. In our model, an increase in the magnitude of private information would be equivalent to an increase in $\theta^b - \theta^g$ and, with $\theta^b < 1$, does not necessarily increase the possibility of rejection. For example, increasing $\theta^b - \theta^g$ and at the same time lowering both θ^b and θ^g can reduce the probability of rejection.

1.4 Variable costs proportional to claims versus premia

When variable administrative costs that are proportional to claims are present the insurer's maximization problem is given by Equations (1)–(3) with $k = 0$ and the first-order conditions are given by Equations (6)–(7). Now suppose instead that the variable administrative costs are proportional to premia. The insurer's maximization problem becomes

$$\max_{\pi^i,\iota^i} \psi[\delta\pi^g - \theta^g\iota^g] + (1-\psi)[\delta\pi^b - \theta^b\iota^b] \tag{41}$$

subject to

$$(PC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, 0, 0) \geq 0, \quad i \in \{g, b\}, \tag{42}$$
$$(IC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, \pi^j, \iota^j) \geq 0, \quad i, j \in \{g, b\},\ i \neq j, \tag{43}$$

where $0 \leq \delta < 1$ reflects the costs. In this case, when Equation (42) binds for the good types and Equation (43) binds for the bad types the optimal menu must satisfy

$$\psi MRS(\theta^g, \pi^g, \iota^g) + (1-\psi)\left[\frac{U_\pi(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)} MRS(\theta^g, \pi^g, \iota^g) + \frac{U_\iota(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)}\right] = \psi\theta^g/\delta, \tag{44}$$
$$MRS(\theta^b, \pi^b, \iota^b) = \theta^b/\delta. \tag{45}$$

Notice that when $\lambda = 1/\delta$ the systems of equations that determine the optimal menus in these two scenarios are equivalent.

1.5 Adding more periods to the quantitative model

In this section we show that period 1 of our 3 period model can easily be replaced with multiple periods in which the young make consumption and savings decisions at annual frequencies. To simplify the analysis we abstract from initial differences in frailty and assume that the entire working-age endowment is received when individuals retire. In our three period model, working-aged individuals face no risks. Thus, the essence of the savings decision of a working-aged person in our model is captured by the following two period consumption-savings problem for an individual where for convenience we assume that the endowment, ω_o is received at the start of the second and final period of life, $V(a_o)$ is the value function of an individual at the point of retirement, and E is the expectations operator.

$$\max_{c_y, a_o} \frac{c_y^{1-\sigma}}{1-\sigma} + \beta EV(a_o)$$
$$s.t.$$
$$c_y + a_o/R = \omega_o.$$

The FONC is:

$$c_y^* = R\beta EV'(a_o).$$

and combining the FONC with the budget constraint yields the following expression for a_o:

$$a_0 = \left(\omega_o - [R\beta EV'(a_o)]^{-1/\sigma}\right) R.$$

Suppose instead that an individual works for J years before retiring. The problem is given by

$$\max_{\{c_j\}_{j=1}^{J},\hat{c}_0} \sum_{j=1}^{J} \gamma^{j-1} \frac{c_j^{1-\sigma}}{1-\sigma} + \gamma^J EV(a_o)$$
$$s.t.$$
$$\sum_{j=1}^{J} c_j/\hat{R}^{j-1} + \hat{a}_o/\hat{R}^J = \omega_o.$$

The FONCs for this problem are:

$$\hat{R}\gamma EV'(a_o) = c_J^{-\sigma},$$
$$c_1^{-\sigma} = c_j^{-\sigma}(\gamma\hat{R})^{j-1}.$$

Using these expressions we can express a_o as:

$$a_0 = \left[\omega_o - \sum_{j=1}^{J} \frac{[\gamma\hat{R}]^{\frac{j-1}{\sigma}}}{\hat{R}^{j-1}} \left((\hat{R}\gamma)^J EV'(a_o)\right)\right] \hat{R}^J. \tag{46}$$

Next note that if there are 40 years of youth then $\gamma = \beta^{1/J}$ and, for a given R, Equation (46) can be solved to find the corresponding value of $\hat{R}$ that delivers the same assets at retirement, a_o, in the two problems.

2 Details of the data work

Our HRS sample is constructed from the 1992 to 2012 waves of HRS and AHEAD. The sample is essentially the same as Braun et al. (2015) and Kopecky and Koreshkova (2014). Beyond adding additional data from 1992, 1994, and 2012, there are a few other changes. There is no censoring at -500 and 500 for asset values near 0. We assign an individual's 1998 weight (or post-1998 mean weight, if their 1998 weight is 0) to pre-1998 waves where their weight is 0. The main definitional novelties/changes are now provided. An individual is retired if his labor earnings are less than \$1500 (in 2000 dollars). An individual is considered to have ever had long-term care insurance if they report having been covered in half or more of their observed waves.

Nursing home event A nursing home event occurs when an individual spends 100 days or more in a nursing home within the approximately two year span between HRS interviews or within the period between their last interview and death. If the individual dies less than 100 days after their last interview, but at the time of their death had been in a nursing home for over 100 days, this also counts as a nursing home event. In the HRS, there are several (sometimes inconsistent) variables that provide information about the number of days spent in a nursing home. From the RAND dataset, we use the total nursing nights over all stays during the wave, as well as the number of days one has been in a nursing home (conditional on being in a nursing home at the time of the interview). This information is also pulled from the exit data, as well as the date of entry to a nursing home, provided the individual died there. Interview and death dates are used when a respondent reports having been continuously in a nursing home since the previous wave. Since the information is sometimes conflicting, and one piece often missing when another observed, a nursing home event is assigned if any of the variables suggest a person met the criteria described above.

Permanent Income To calculate permanent income, first sum the household heads social security and pension income and average this over all waves in which the household head is retired. The cumulative distribution of this average is defined as the permanent income, which ranges from 0 to 1. For singles, the household head is the respondent, and for couples it is the male.

Wealth We use the wealth variable ATOTA which is the sum of the value of owned real estate (including primary residence), vehicles, businesses, IRS/Keogh accounts, stocks, bonds, checking/savings accounts, CDs, treasury bills and "other savings and assets" less any debt reported.

2.1 Rejections

Many applications for LTCI are rejected. Murtaugh et al. (1995) in one of the earliest analyses of LTCI underwriting estimates that 12–23% of 65 year olds, if they applied, would be rejected by insurers because of poor health. Their estimates are based on the National Mortality Followback Survey. Since their analysis, underwriting standards in the LTCI market have become more strict. We estimate rejection rates of between 36% and 56% for 55–65 year olds by applying underwriting guidelines from Genworth and Mutual of Omaha to a sample of HRS individuals.

To understand how we arrive at these figures, it is helpful to explain how LTCI underwriting works. Underwriting occurs in two stages. In the first stage, individuals are queried about their prior LTC events, pre-existing health conditions, current physical and mental capabilities, and lifestyle. Some common questions include: Do you require human assistance to perform any of your activities of daily living? Are you currently receiving home health care or have you recently been in a NH? Have you ever been diagnosed with or consulted a medical professional for the following: a long list of diseases that includes diabetes, memory loss, cancer, mental illness, heart disease? Do you currently use or need any of the following: wheelchair, walker, cane, oxygen? Do you currently receive disability benefits, social security disability benefits, or Medicaid?[1] A positive answer to any one of these questions is sufficient for the insurers to reject applicants before they have even submitted a formal application. Many of these same questions are asked to HRS participants. As Table 1 shows, the fraction

[1]Source: 2010 Report on the Actuarial Marketing and Legal Analyses of the Class Program

Table 1: Percentage of HRS respondents who would answer "Yes" to at least one LTCI prescreening question.

	Age		
	55–56	60–61	65–66
All	41.8	43.7	49.5
Top Half of Wealth Distribution Only	30.8	33.6	39.3

Data source: Authors' calculations using our HRS sample.

of individuals in our HRS sample who would respond affirmatively to at least one question is high and ranges from 40.5–to 49.6% depending on age. Rejection rates are also high in the top half of the wealth distribution ranging from 30.8% –to 39.3%. Question 3 pertaining to previously diagnosed diseases received the highest frequency of positive responses. If we are conservative and omit question 3 the prescreening declination rate ranges from 17.5–22.5% for all individuals and from 10.0% –12.1% for individuals in the top half of the wealth distribution.

If applicants pass the first stage, they are invited to make a formal application. Medical records and blood and urine samples are collected and the applicants cognitive skills are tested. One in five formal applications are denied coverage based on industry surveys (see Thau et al., 2014). Assuming a 20% rejection rate for the second round, the overall rejection rate is 55.6% for 55–66 years old in our HRS sample using all questions and 35.9% if question three is omitted. For individuals in the top half of the wealth distribution the rejection rates are 47.7% and 28.0% respectively.

2.2 Construction of the Frailty Index

Table 2 lists the variables we used to construct the frailty index for HRS respondents. The choice of these variables is based on Genworth and Mutual of Omaha LTCI underwriting guidelines. To construct the frailty index, first sum the variables listed in the first column of Table 2, assigning each a value according to the second column. Then divide this sum by the total number of variables observed for the individual in the year, as long as the total includes 30 or more variables. The construction of this frailty index mostly follows the guidelines laid out in Searle et al. (2008), and uses a set of HRS variables similar to the index created in Yang and Lee (2009). There are a couple of differences however. Primarily, a few variables that do not necessarily increase with age (e.g. drinking $>$ 15 drinks per week and smoking) were included. Also, cognitive tests are broken into parts which each count as separate variables, essentially increasing their weight in the index relative to Searle et al. (2008), which uses only a single variable for cognitive impairment. Nevertheless, our frailty distribution still closely resembles those of frailty indices used in other papers.

Table 2: Health Variables for Frailty Index Construction

Variable	Value
Some difficulty with ADL/IADLs:	
Eating	Yes=1, No=0
Dressing	Yes=1, No=0
Getting in/out of bed	Yes=1, No=0
Using the toilet	Yes=1, No=0
Bathing/shower	Yes=1, No=0
Walking across room	Yes=1, No=0
Walking several blocks	Yes=1, No=0
Using the telephone	Yes=1, No=0
Managing money	Yes=1, No=0
Shopping for groceries	Yes=1, No=0
Preparing meals	Yes=1, No=0
Getting up from chair	Yes=1, No=0
Stooping/kneeling/crouching	Yes=1, No=0
Lift/carry 10 lbs	Yes=1, No=0
Using a map	Yes=1, No=0
Taking medications	Yes=1, No=0
Climbing 1 flight of stairs	Yes=1, No=0
Picking up a dime	Yes=1, No=0
Reaching/ extending arms up	Yes=1, No=0
Pushing/pulling large objects	Yes=1, No=0
Cognitive Impairment:	
Immediate Word Recall	+.1 for each word not recalled (10 total)*
Delayed Word Recall	+.1 for each word not recalled (10 total)*
Serial 7 Test	+.2 for each incorrect substraction (5 total)
Backwards Counting	Failed test=1, 2nd attempt = .5, 1st attempt = 0
Identifying obejcts & Pres/VP	.25 for each incorrect answer (4 total)
Identifying date	.25 for each incorrect answer (4 total)
Ever had one of following conditions:	
High Blood Pressure	Yes=1, No=0
Diabetes	Yes=1, No=0
Cancer	Yes=1, No=0
Lung disease	Yes=1, No=0
Heart disease	Yes=1, No=0
Stroke	Yes=1, No=0
Psychological problems	Yes=1, No=0
Arthritis	Yes=1, No=0
BMI ≥ 30	Yes=1, No=0
Drinks 15+ alcoholic drinks per week	Yes=1, No=0
Smokes Now	Yes=1, No=0
Has smoked ever	Yes=1, No=0

*For the 1994 HRS cohort, 40 questions were asked (instead of 20) for word recall. In this year, each missed question receives weight 0.05.

Table 3: LTCI take-up rates by wealth and frailty for married and not married individuals

Frailty Quintile	Married: Wealth Quintile 1	2	3	4	5	Not Married: Wealth Quintile 1	2	3	4	5
1	0.011	0.044	0.095	0.151	0.242	0.010	0.093	0.108	0.134	0.205
2	0.031	0.057	0.119	0.156	0.219	0.012	0.017	0.047	0.163	0.175
3	0.011	0.027	0.084	0.141	0.212	0.031	0.038	0.091	0.092	0.174
4	0.014	0.036	0.068	0.098	0.178	0.016	0.021	0.069	0.145	0.127
5	0.026	0.026	0.040	0.095	0.076	0.007	0.032	0.066	0.122	0.145

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. Data source: 62–72 year olds in our HRS sample.

2.3 Evidence of private information

Hendren (2013) finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We restrict the sample to individuals ages 65–80. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a subsample of this sample consisting of individuals who would likely be rejected by insurers. This sample includes individuals who have any ADL/IADL restriction, past stroke, or past nursing or home care. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.003 at the 10 year horizon and 0.169 at the 6 year. If all individuals above age 80 are included in the reject sample as well the p-values at both horizons are less than 0.000. For a sample of individuals who would likely not be rejected we are unable to find evidence of private information. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.210 at the 10 year horizon and 0.172 at the 6 year.

2.4 LTCI take-up rate patterns controlling for family status

Tables 3 and 4 shows LTCI take-up rates of by frailty and wealth quintiles for married versus single individuals and individuals with and without children. The general pattern of take-up rates by frailty and wealth are robust to controlling for martial status and children. LTCI take-up rates increase with wealth and decline with frailty for both married and single individuals and for both individuals with and those without children. Comparing the levels of take-up rates across married and single individuals shows that in many wealth and frailty quintiles there is no systematic difference between them. The only discernible differences between individuals with and without children are that wealthy individuals without children, those in quintiles 4 and 5 of the wealth distribution, tend to have slightly higher take-up rates then those with children.

Table 4: LTCI take-up rates by wealth and frailty for individuals with and without children

	Have Children					Do Not Have Children				
Frailty	Wealth Quintile					Wealth Quintile				
Quintile	1	2	3	4	5	1	2	3	4	5
1	0.012	0.058	0.100	0.146	0.221	0.000	0.059	0.076	0.186	0.337
2	0.017	0.051	0.103	0.153	0.196	0.061	0.000	0.064	0.238	0.314
3	0.022	0.033	0.085	0.132	0.202	0.022	0.000	0.088	0.071	0.214
4	0.017	0.033	0.067	0.106	0.142	0.000	0.021	0.130	0.178	0.271
5	0.017	0.025	0.043	0.101	0.101	0.001	0.021	0.239	0.132	0.173

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. Data source: 62–72 year olds in our HRS sample.

2.5 Description of the auxiliary simulation model

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups, we use an auxiliary simulation model similar to that in Hurd et al. (2013). First, using a multinomial logit, we estimate transition probabilities between four states that we observe in the HRS: alive and dead, each with and without a nursing home event in the last two years. These probabilities depend on age, PE, NH event status, and frailty, including polynomials and interactions of these variables. Specifically, age is modeled as a cubic function, frailty as a quadratic, and the others are both linear. Interactions include age with each of the other first-order terms, as well as frailty with PE. We also simulate lifetime frailty paths because we need them since, in contrast to Hurd et al. (2013), we include frailty in the multinomial logit. This is done using estimates from a fixed effects regression of frailty on lagged frailty, age, and age squared.

Simulations begin at age 67. To get the initial distribution of explanatory variables, we first average frailty and population weights across all observations at which an individual is between ages 62-72. PE and the estimated fixed effect are constant within individuals. The initial distribution then draws 500,000 times from this person-level weighted distribution. The model simulates two-year transitions, following the structure of the HRS data, and assigns age of death by randomly choosing an age between their last living wave and the death wave.[2]

3 Computation

Computing an equilibrium in our model is subtle because Medicaid NH benefits are means-tested and Medicaid is a secondary payer of NH benefits. Individual saving policies exhibit

[2]Note that a half year is added to death age to account for the fact that reported ages are the floor of a respondent's continuous age. Nursing home entry ages are similarly assigned, but we add only 0.2 years due to the 100 day requirement of a nursing home event. They are also upwardly bound by death age when both occur in the same wave.

jumps and the demand for private insurance interacts in subtle ways with $q(\kappa)$, the distribution of consumption demand shocks.

We start by discretizing the endowment and frailty distributions. The number of grid points for endowments $\mathbf{w}$ is $ny = 101$ and frailty takes on $nf = 5$ grid points. The consumption demand shock κ is also discretized: $nk = 50$.

The specific algorithm for computing an equilibrium proceeds as follows. First, we guess values for profits (which gives us dividends) and taxes and then we iterate over profits and taxes until profits converge and taxes satisfy the government budget constraint. In each iteration, we have to solve for allocations, contracts, and profits for each combination of endowments and frailty in the discretized state space. For each point in the discertized state space $(\mathbf{w}_s, f_j)$, $s = 1, nw$ and $j = 1, nf$, we guess a level of savings: $\hat{a}_{f_j,\mathbf{w}_s}$. Given $\hat{a}_{f_j,\mathbf{w}_s}$, we then solve for the optimal contracts as follows. The optimal contract for a risk group depends on which individuals of observable type $(\mathbf{w}_s, f_j)$ qualify for Medicaid if they incur the NH shock. Thus, it depends on the specific combinations of the κ shock and the private type $i \in \{g, b\}$ that imply that an individual qualifies for Medicaid. Because of the non-convexities introduced by Medicaid the Kuhn-Tucker conditions of the insurer's problem are not sufficient. However, if one first assumes a distribution of individuals across Medicaid, then a contract satisfying the Kuhn-Tucker conditions is sufficient. So we solve for the optimal contract for all feasible combinations of individuals with different κ's and i's receiving Medicaid. The number of cases that has to be considered is large but it can reduced by the noting that for a given value of κ if a bad type is on Medicaid the good type is also on Medicaid by the single-crossing property and that if a type i qualifies for Medicaid for a value of κ he will also qualify for Medicaid for all larger values of κ.

To solve for the optimal contracts for each Medicaid distribution, first we solve for the optimal pooling contract. Second, we check to see if an optimal separating menu exists. The contract of type g under the optimal separating menu is the same as the optimal pooling contract. So we fix the good type's contract at the optimal pooling one and solve for the optimal separating contract of the bad type (if it exists).[3] The optimal contract for observable type $(\mathbf{w}_s, f_j)$ under the current guess for savings, $\hat{a}_{f_j,\mathbf{w}_s}$ is then the one that maximizes the insurer's profits. Finally, we iterate over savings until we find the value of savings that maximizes expected lifetime utility.

4 Additional Calibration Details

Table 5 list the values of many of the model parameters. The survival probabilities of each frailty and PE quintile in the quantitative model are shown in the left panel of Figure 2. The right panel shows the mean lifetime NH entry probability conditional on surviving for each frailty/PE quintile combination in the model. These NH entry probabilities in the model match those in the data because we parametrized the model to reproduce the survival probabilities in the left panel and the unconditional NH entry probabilities in Figure 4 in the paper.

[3] If a separating menu doesn't exist it means one of the inequality constraints is violated.

Table 5: Model Parameters

Description	Parameter	Value
Risk aversion coefficient	σ	2
Preference discount factor	β	0.94
Retirement preference discount factor	α	0.20
Interest rate (annualized)	r	0.00
Frailty distribution	h	BETA(1.54,6.30)
Young endowment distribution	w_y	$\ln(w_y) \sim \mathcal{N}(-0.32, 0.64)$
Copula parameter	ρ_{f,w_y}	-0.29
Demand shock distribution	κ	$1-\kappa \sim$ truncated log-normal
Demand shock mean	μ_κ	0.6
Demand shock standard deviation	σ_κ	0.071
Fraction of good types	ψ	0.709
Nursing home cost	m	0.0931
Insurer's variable cost of paying claims	λ	1.195
Insurer's fixed cost of paying claims	k	0.019
Medicaid consumption floor	$\underline{c}_{NH}$	0.01855
Welfare consumption floor	$\underline{c}_o$	0.01855

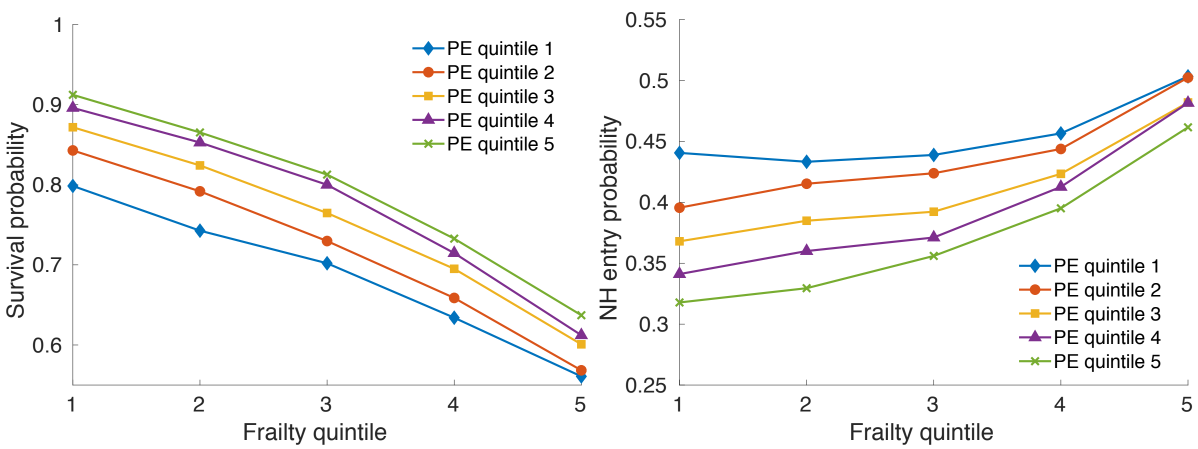


Figure 2: The probability of surviving to age 80 or until experiencing a NH stay (left panel) and the probability that a 65-year old will enter a NH conditional on surviving to age 80 (right panel) by frailty and PE quintile. The probabilities are based on our auxiliary simulation model which is estimated using HRS data. NH entry probabilities are for a NH stay of at least 100 days.

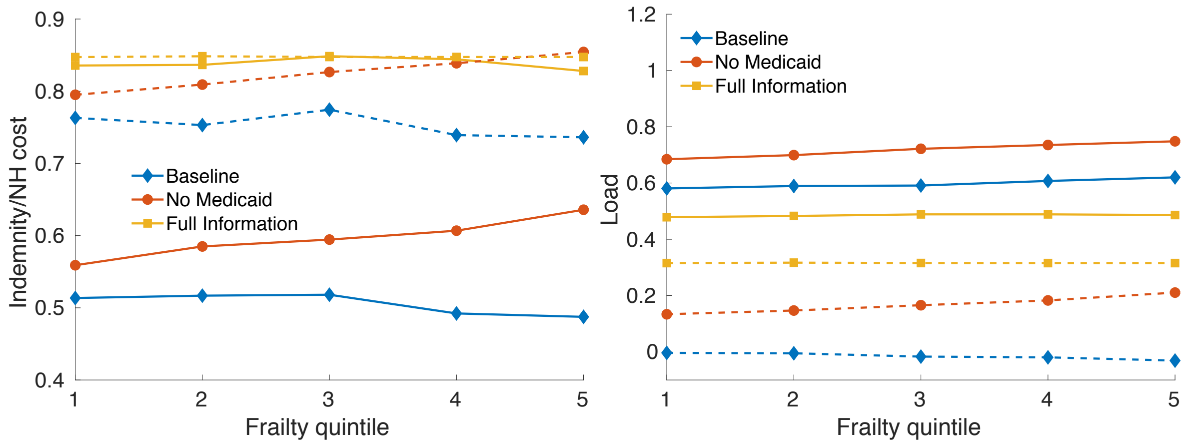


Figure 3: Insurance coverage and loads by frailty quintile.
The left panel reports LTCI indemnities relative to medical costs of a NH stay and the right panel reports loads for the two private information types: good risks (solid lines) and bad risks (dashed lines). Three economies are reported: Baseline, Full Information and No Medicaid.

Determination of nursing home cost m. We estimate the average medical and nursing expense component of NH costs as follows. First, we use data from the Minnesota Office of the Legislative Auditor (1995) which provides a breakdown of SNF and RCC costs for 5 midwest states in 1994. We adjust each cost in the breakdown using either the CPI or the medical CPI to create a cost breakdown for the year 2000. Then we calculate the share of costs due to medical and nursing expenses and the share due to room and board for each state and average them across states using state population weights. The population weights are taken from the 2000 U.S. Census. We find that, on average, 76% of SNF and RCC costs are due to medical and nursing expenses and 24% are room and board. Next, we obtain estimates of the average annual total costs of SNF and RCC stays in the U.S. in 2000 of $60,000 and $28,099, respectively, from Stewart et al. (2009). Using these and the shares, we calculate the average annual cost of the medical and nursing expense component of each type of stay. Finally, we average the annual cost of the medical and nursing expense component across NH and RCC stays using data from Spillman and Black (2015) on the fraction of individuals in residential care who are in RCC's versus SNF's. We obtain an average medical and nursing expense component of residential LTC costs of $32,844 per annum in year 2000. Braun et al. (2015) estimate that the average duration of NH stays that exceed 90 days is 3.25 years. Medicare provides NH benefits for up to the first 100 days. To account for this, we subtract 100 days resulting in an average benefit period of 2.976 years. Multiplying the annual cost by the average benefit period yields total medical and nursing costs of a NH stay of $97,743 or a value of $m = 0.0931$ when scaled by average lifetime earnings.

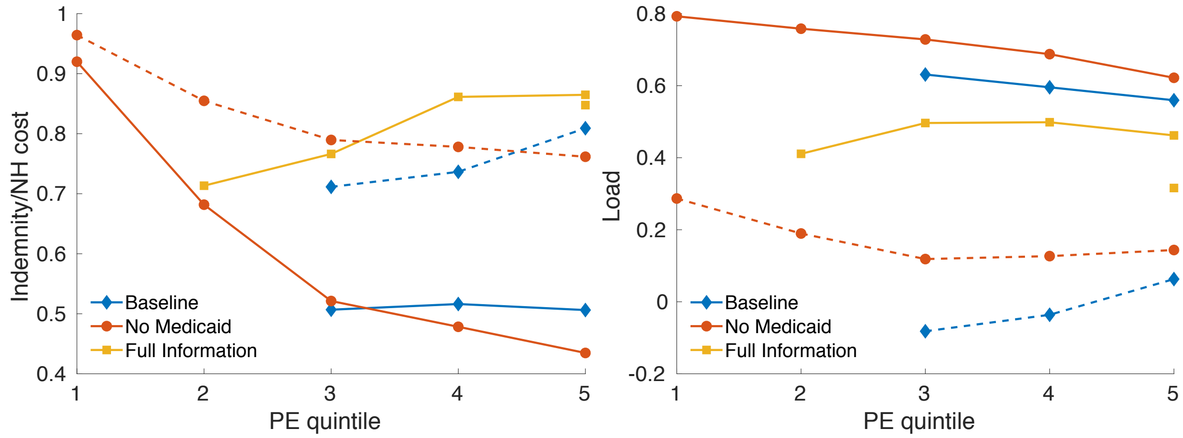


Figure 4: Insurance coverage and loads by PE quintile.
The left panel reports indemnities relative to the medical cost of a NH stay and the right panel reports loads for the two private information types: good risks (solid lines) and bad risks (dashed lines). Three economies are reported: Baseline, Full Information and No Medicaid.

5 Additional Results

5.1 Pricing and Coverage in Baseline and Other Economies

Figures 3 and 4 show how Medicaid and adverse selection influence pricing and coverage at alternative frailty and PE levels. Removing private information increases the coverage of both types relative to coverage in the Baseline and reduces the variation in loads. Reducing the scale of Medicaid also increases the level of coverage for both private information types at each frailty quintile. However, the loads are also higher. Notice also that coverage increases monotonically in frailty in the No Medicaid economy.

Figure 4 reports how coverage and loads vary by PE quintile for the same three scenarios. In the Full Information economy bad risks in PE quintiles 1–4 do not get positive private insurance. Coverage of good risks in increasing in PE and higher than in the Baseline. Loads on good risks are lower than in Baseline and humped-shape in PE. In the No Medicaid economy, as PE increases, bad and good risk types experience lower coverage and slightly declining loads. Reducing the Medicaid NH benefit floor has a very big impact on the poor. Their demand for LTCI is inelastic and, as a result, they now face the highest loads but also receive the most coverage.

5.2 Robustness Analysis

Size of the Medicaid Consumption Floor Our Baseline parameterization of the model uses the same Medicaid consumption floor as Brown and Finkelstein (2008). They note that their results are sensitive to the size of this parameter and other empirical work has sometimes used higher consumption floors. To assess the robustness of our conclusions to the scale of Medicaid we recalibrated the model positing a Medicaid consumption floor that is 1.76 times larger than the value in the Baseline economy. This value is at the high end of previous estimates (see Kopecky and Koreshkova (2014) for a summary of consumption floor

Table 6: Robustness: Rejection rates in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information Economies with Higher Consumption Floors.

Scenario Description	Baseline	No Admin. Costs $\lambda = 1$, $k = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information $\theta^i_{f,\mathbf{w}}$ public
Average	90.1	39.0	9.9	48.3
By PE Quintile				
1	100	100	15.4	100
2	100	83.3	0.0	99.8
3	82.5	0.0	0.0	31.8
4	86.8	0.0	0.0	0.0
5	75.8	0.0	0.18	50.0
By receiving Medicaid NH benefits conditional on surviving				
Would	44.1	35.7	3.1	41.1
Would not	44.9	1.0	0.0	15.2

Rejection rates are percentage of individuals who are only offered a single contract of $(0, 0)$ by the insurer. Note that for PE quintiles that the figures are expressed as a percentage of individuals in that quintile. However, the bottom two rows of the table are a decomposition of the average rejection rate for that economy.

values). Table 6 indicates that Medicaid now has a bigger impact on producing rejections among affluent households. Rejection rates are only 18% in wealth quintile 5 if Medicaid is removed. However, administrative costs and private information continue to be very important among more affluent individuals. Removing either friction still has a very large impact on rejection rates in PE quintiles 3-5. Rejection rates fall to zero in these groups if administrative costs are removed and they fall by 50% or more if private information is removed. In addition, supply-side frictions continue be responsible for the large fraction of individuals who pay for NH expenses out-of-pocket in the Baseline.

How well can a model do that abstracts from private information? We have found that the Full Information economy has very high LTCI take-up rates among higher wealth quintiles and produces an incorrect pattern of take-up rates by frailty in wealth quintiles 4 and 5. Is there a way to remedy these issues with the Full Information economy by recalibrating it? To explore this possibility we recalibrated the Full Information economy so that it reproduces the average LTCI take-up rate by increasing fixed and variable administrative costs in a proportionate fashion. The resulting magnitude of the administrative costs increased from 32.6% of premium to 49% of premium. The Full Information economy with higher administrative costs continues to have problems reproducing the pattern of LTCI take-up rates by wealth and frailty quintile. For instance, LTCI take-up rates in wealth quintile 5 are now zero. From the perspective of this group these administrative costs are so high that they prefer to self-insure NH risk. LTCI take-up rates in wealth quintile 4 are positive. However, they are not declining in frailty as occurs in our HRS data (See

Table 2 in the paper). We also explored lowering the LTCI take-up rates in this economy by varying the θ's. However, we could not generate enough variation in the θ's to reproduce the average level of LTCI take-up we see in the data nor did varying the θ's help us reproduce the empirical pattern of LTCI take-up rates by frailty in the top two wealth quintiles.

How well can a model do that abstracts from administrative costs? We have also investigated whether a version of the model with no administrative costs could hit our calibration targets. It is also a challenge for the model to reproduce low LTCI take-up rates in high wealth quintiles when administrative costs are zero. Consider, for instance, the group in wealth and frailty quintile 5. When administrative costs are set to zero the LTCI take-up rate for this group increases from 0.118 to 0.99. if the nursing home entry rate for bad risks is increased to one ($\theta^b = 1$) the LTCI take-up rate falls to 0.96. This model also predicts a high LTCI take-up rate for this group if assume that one half of all individuals are bad risks $\psi = 0.5$ and have NH entry probabilities of one. The model produces a LTCI take-up rate of 0.44 while the LTCI take-up rate for this group in our dataset is only 0.104.

Taken together these results suggest that both private information and administrative costs are required if the model is to produce LTCI take-up rates that have the same magnitude and pattern across different wealth and frailty quintiles as our data.

Insurers do pay out at the other end. One reason that has been offered for low LTCI take-up rates is that people are concerned that insurers will come up with reasons for not paying out at the time the NH event occurs (see, for instance, Duhigg (2005)). However, survey evidence suggests that most individuals are happy with the claims filing experience. A survey conducted in 2015–2016 by LifePlans Inc., a service provider for insurers, found that 78 percent of claimants found it easy to file a claim.[4] Only 6% had a disagreement with their company about coverage and disagreements in a majority of cases were resolved in favor of the policy holder. Taken together they find that only 2% claims filers find themselves in a situation where they disagree with their insurer and the problem is not resolved to their satisfaction. Another survey commissioned by the U.S. Department of Health and Human Services in 2007 produced similar results (See U.S. Department of Health and Human Services (2007)). It finds that benefits were approved for 95.7% of respondents filing claims and that of those initially denied benefits more than half subsequently received benefits in the ensuing 12 month period.

Multiple sources of private information and heterogeneous preference discount rates. We have found that our model with a single source of private information can account for a broad range of empirical regularities in the U.S. LCTI market including the correlation puzzle. This is not to say that insurers do not have to contend with private preference heterogeneity in risk or preference discount rates in this market. Our analysis suggests that these considerations may not be of first order importance to issuers given the institutional features in the U.S. market that we have modeled. To provide a specific example we multiple sources of private information may not be of central importance here, consider private differences in preference discount rates. Higher preference discount rates among

[4]See Lifeplans, Inc. (2016).

frail could possibly help account for rejections among poorer individuals but, Medicaid is very already very effective in producing rejections among poorer and even middle class individuals. So it is not clear that there is a need to appeal to private differences in discount rates to produce rejections in these groups. It is also not clear that modeling a positive correlation between frailty and privately observed preference discount rates would be help us in accounting for rejections among wealthy frail individuals. The first order implication of a high preference discount rate is to save less and consume more and one would thus expect that frail individuals in high wealth quintiles are reasonably patient.

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